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Sub report 11:
Cohesive Sediments Dimensional Analysis.

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Nautical Bottom Sediment Research

Sub report 11: Cohesive Sediments Dimensional Analysis

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2.0	19/06/2012	Substantive revision	Van Hoestenberghe,T.; Claeys S.
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4.0	31/01/2013	Substantive revision & Internal Costumer revision	Vanlede, J.
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Abstract

The dimensional analysis technique was applied in order to determine possible relationships between the measured parameters in the STT and to identify possible inconsistencies in the measurements.

The present dimensional analysis focuses only in the sedimentation and consolidation processes.

Relationships between parameters could be used to identify inconsistencies for measured pore pressure, effective stresses and density values. Recommendations are suggested for sampling points and measuring methods.

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1 Introduction

Several measurements were performed in the Sediment Test Tank (STT) for the study of the evolution of sediment parameters. The dimensional analysis technique was applied in order to determine possible relationships between the parameters. In addition, this technique is useful in the sense that it would be possible to identify possible inconsistencies in the measurements. Hence, further filtering could be applied and improved for further measurements.

The present dimensional analysis focuses only on the sedimentation and consolidation processes. It is a first phase of the application of this technique. The next phase will include additional processes, such as rheology. Also parameters that influence the rheological parameters, such as organic matter content, dissolved oxygen and conductivity, can be included in the next phase.

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2 Parameters

The application of the dimensional analysis technique starts with selecting the parameters to be analysed. Since the present analysis focusses on the sedimentation and consolidation processes only, the parameters to be analysed are taken from the mass balance equation (see Eq. 13 of Heredia et al.; 2012), these parameters are:

• $\Delta\rho$	excess density	[kg/m^3] (see paragraph 5.1)
• w_{s0}	Stoke's settling velocity	[m/s]
• g	gravity constant	[m/s^2]
• σ'	effective stress	[Pa]
• z	vertical coordinate	[m]
• t	time	[s]
• ρ_w	water density	[kg/m^3]

It has to be noticed that the mentioned equation uses the settling rate (w_0) which is the average velocity associated with the solids flux, being different from the Stoke's settling velocity. The settling rate can be determined from the measured density profiles (Toorman and Leurer, 2000). However, this analysis is a first approach. For a more detailed approach, a filtering technique needs to be applied in order to filter-out possible inconsistencies in the measured densities. At this moment, this task is not yet performed. Hence, as a first approach, the dimensional analysis uses the Stoke's settling velocity. This approach should be modified in the future when the filtered data of densities becomes available.

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3 Dimensional analysis

The main aim behind the application of the dimensional analysis technique, is the attempt of approaching the mass balance equation for sedimentation and consolidation. Besides, the application of the dimensional analysis tries to reproduce the observations reported in Toorman and Huysentruyt (1997).

The dimensional analysis is based on the Pi Buckingham's theorem, where different relationships of the listed parameters can be produced. It is assumed (according to the employed technique) that these relationships are non-dimensional (Hughes, 1993). The theorem establishes that a dimensionally homogeneous equation involving n variables, such as:

$$x_1 = f\left(x_2, x_3, x_4, ..., x_n\right) \tag{1}$$

is an equation in which the dimensions of the variable on the left side of the equality match the dimensions of any term that stands by itself on the right side of the equality. The theorem, states also that any such equation can be rearranged into a new equation expressed in terms of dimensionless products (pi terms):

$$\Pi_1 = \Psi\left(\Pi_2, \Pi_3, \Pi_4, ..., \Pi_{n-r}\right) \tag{2}$$

The required number of pi terms is less than the number of original variables by the amount r, where r is the number of fundamental dimensions contained in the original variables. If the original n variables are independent, then the $n-r$ dimensionless products will also be independent provided all original variables appear at least once in one of the dimensionless products.

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4 Dimensionless products

The dimensionless products are the result of the application of the dimensional analysis, these products will be the basis for the relationships to be determined. Following the criteria of the Buckingham's theorem, four dimensionless products are defined: $n - r = 4$; where the number of independent variables is 7 and the number of fundamental dimensions is 3. Applying this technique allows several possibilities for the determination of dimensionless products, the final selection of the most relevant products is based on the physical background of the studied process. In this case the consolidation, where: the height, the excess density and the effective stress are the most relevant parameters and should appear in the final products. Then, the final list of the dimensionless products is presented below:

First product:

$$\Pi_1 = \frac{\Delta\rho w_{s0}}{(zg)^{1/2}\rho_w}$$

Second product:

$$\Pi_2 = \frac{w_{s0}}{(zg)^{1/2}}$$

Third product:

$$\Pi_3 = \frac{\sigma'}{z^3 g\rho_w}$$

Fourth product:

$$\Pi_4 = \frac{\sigma' w_{s0}}{(zg)^{3/2} z^2 \rho_w}$$

Leading to the following definition:

$$\frac{\Delta\rho w_{s0}}{(zg)^{1/2}\rho_w} = f\left(\frac{w_{s0}}{(zg)^{1/2}}, \frac{\sigma'}{z^3 g\rho_w}, \frac{\sigma' w_{s0}}{(zg)^{3/2} z^2 \rho_w}\right) \qquad (3)$$

However, it is possible to define additional dimensionless products by the combination of the existing ones. Any combination of these products will also lead to dimensionless new products. Hence, an additional product is proposed:

Fifth product:

$$\Pi_5 = \frac{\Delta\rho}{\rho_w}$$

Even considering that time is used in the application of the dimensional analysis, this parameter does not appear in the final dimensionless products. The reason lies in the fact that larger numerical values are obtained if time appears in the dimensionless products (up to the order of 1E+11), making it difficult to achieve a more feasible interpretation of the obtained relationships.

Figure 1 depicts the relationship of the first and second dimensionless products when the time parameter is used. Larger values for the first product are produced when time is used, it is advisable to avoid the use of larger values in this analysis.

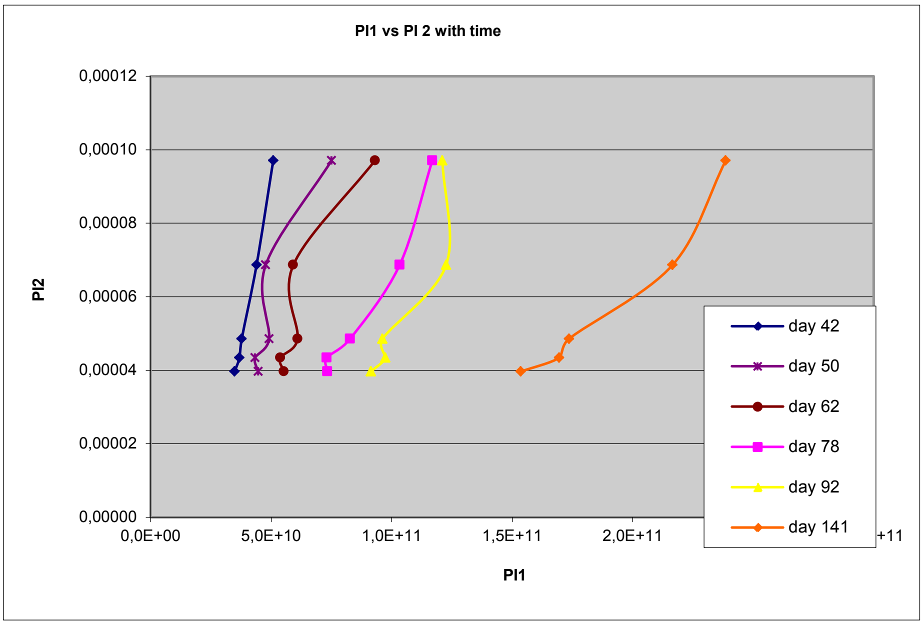

Figure 1 : Relationships of the First and the second dimensionless products when time parameter is included

5 Values of the parameters used for the dimensional analysis

From the list of parameters presented in §2, three parameters are assumed to be constant for the further calculations. These parameters are approximations and are given below:

- $g = 9.81\, m/s^2$ gravity
- $w_{s0} = 0.000135978\, m/s$ Stoke's settling velocity (calculated from Stoke's eq.)
- $\rho_w = 1000.00\ kg/m^3$ water density

The rest of the parameter values correspond to measurements and calculated values (based on the measurements) performed in the STT.

5.1 Excess density

The excess density is defined as the difference between the mixture (bulk) density and the water density, this parameter is commonly used to design the sediment concentration in numerical modelling and is given by:

$$\Delta\rho = \rho - \rho_w \tag{4}$$

where ρ is the density of the mixture (g/cm³). This parameter is measured in the STT experiment. The measurements were performed at several depths (records indicate at every centimetre) along the vertical and for several days after the release of sediments. However, the density measurements started after 6 days after the release of the sediments into the STT. In addition, no measurements were made at the bottom of the tank, the lowest level for the measurements corresponds to 0.20m above the bottom.

As a result, the measured densities were used to calculate the excess density assuming the constant value of the water density.

5.2 Effective stress

The effective stress is defined as the difference of the total stress (in a saturated soil) with the pore pressure, being given by:

$$\sigma' = \sigma - p \tag{5}$$

where σ denotes the total stress and p denotes the pore pressure. The total stress is calculated from the measured densities of the mixture (saturated soil). Since the densities were measured at several locations, the total stress can be calculated for several depths as well. However, the pore pressure was measured only at six locations along the vertical (0cm, 20cm, 40cm, 80cm 100cm and 120cm from the bottom).

Leading to a limitation in the number of vertical locations for the calculation of the effective stress. Moreover, considering that the density was not measured at the bottom, the number of vertical locations where the effective stress can be calculated is even less (five points).

On the other hand, difficulties have been observed during the measuring of the pore pressure. It is explained in the Technical Report of the STT experiment (Van Hoestenberghe et al.; 2011). There, it is observed that the pore pressure presents some fluctuations after day 141, making a proper calculation of the effective stress difficult to do. Therefore, only a time interval is selected where no fluctuations are observed for days: 42, 50, 62, 78, 92 and 141.

5.3 Depth (vertical coordinate)

As explained before, the depth will be limited to the number of points where the pore pressure was measured: 5 depths measured from the bottom. Therefore, the dimensional analysis is applied to five depths and six days.

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6 Results of the dimensional analysis

The results of the dimensional analysis focus on the different relationships of the obtained dimensionless products, several combinations are possible. However, here the most relevant according the observations reported in Toorman and Huystentruyt (1997) are presented.

6.1 First product vs. second product:

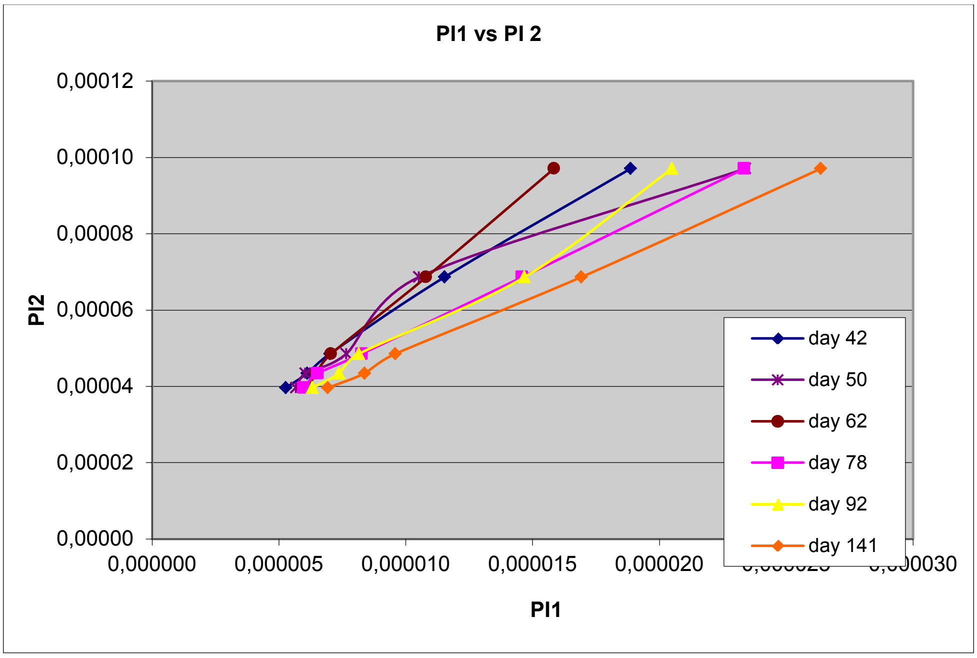

Figure 2 : Relationships of the First and the second dimensionless products

This relationship is the starting point for this analysis, since it presents the relationship of the excess density with the water density and its evolution along the time. It is clear that densities are varying along the time and depth. For different depths, the relationship of the excess density and the water density is increasing according to the consolidation process. In addition, this relationship also depicts possible inconsistencies with the measurement data. In this case, the measured densities for days 50, 62 and 92 do not follow the general behaviour, showing clearly that these measurements have to be verified.

Notice that water density in this case is used as a constant value. However, this analysis is showing clearly that water density plays role in the evolution of densities and should be considered in cases when water densities are varying (e.g. estuaries or intrusion of salinity).

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6.2 First product vs. fifth product

This relationship has to be taken as a complementary information as well, since both products show the excess density. However, it is possible to draw some conclusions after analysing this graph (see Figure 3).

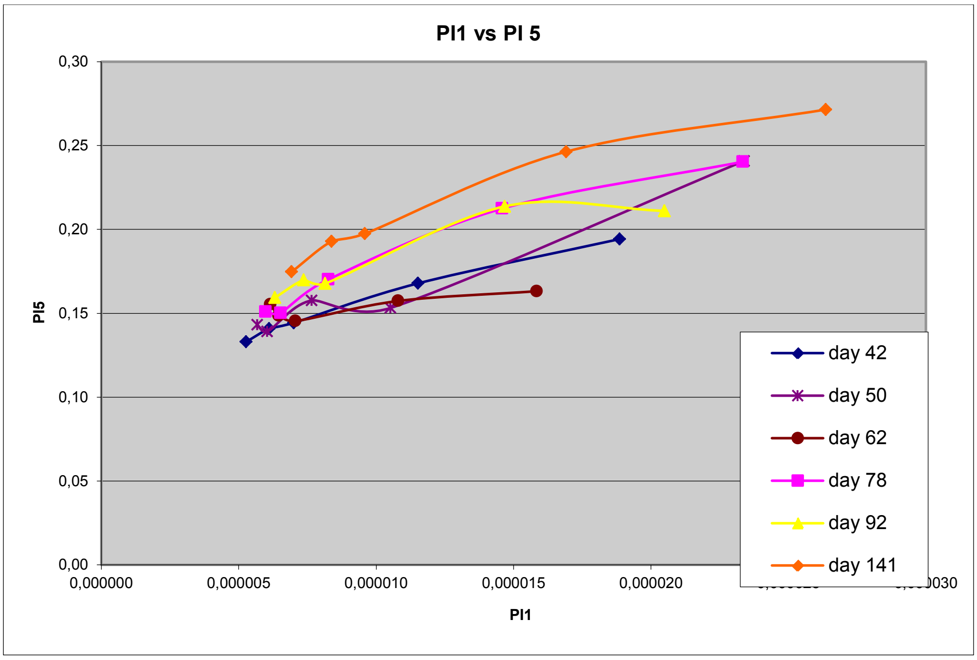

Figure 3 : Relationships of the First and the fifth dimensionless products

This plot presents a family of curves that increase with time. This makes sense since the density increases with time.

6.3 Second product vs. fifth product

This relationship is complementary to the previous one, the same behaviour is observed, the density increase along the time and with the depth.

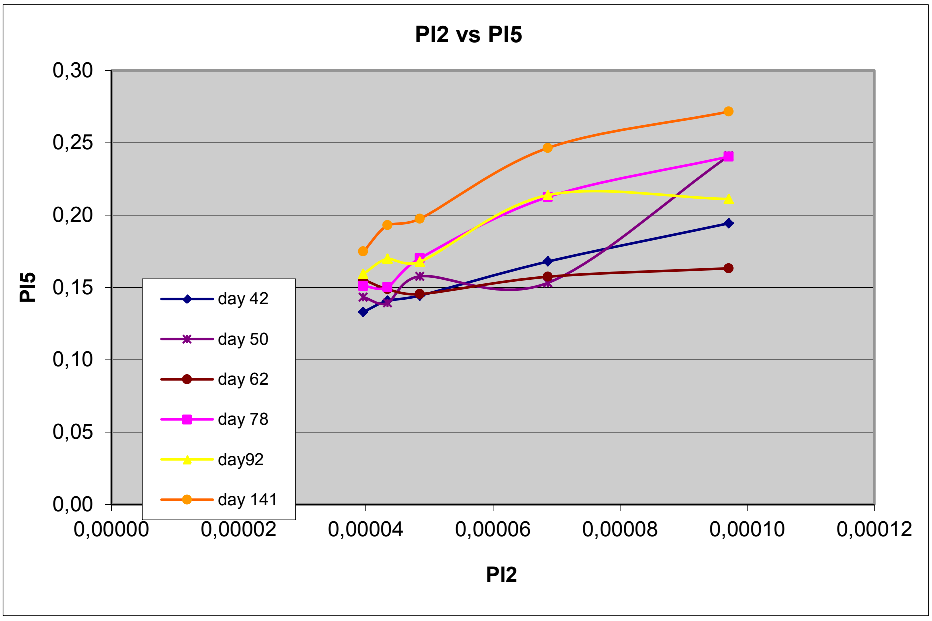

Figure 4 : Relationships of the second and the fifth dimensionless products

6.4 First product vs. third product

This relationship is used to reproduce the observations reported by Toorman and Huysentruyt (1997), where the effective stress is a function of the density and other parameters. In this case, the effective stress is a function of the depth, the water density and the gravity. It is clear that all of these parameters are interrelated. When the effective stress increases as a result of the consolidation process, the density will increase as well. These relationships show an asymptotic behaviour. For the lowest values of the density (close to the interface) the effective stress is almost constant and close to zero, increasing abruptly along the depth, showing higher values close to the water surface. Close to the bottom, the graph show an asymptotic behaviour as well, when the effective stress increases, the density will reach a 'maximum' value.

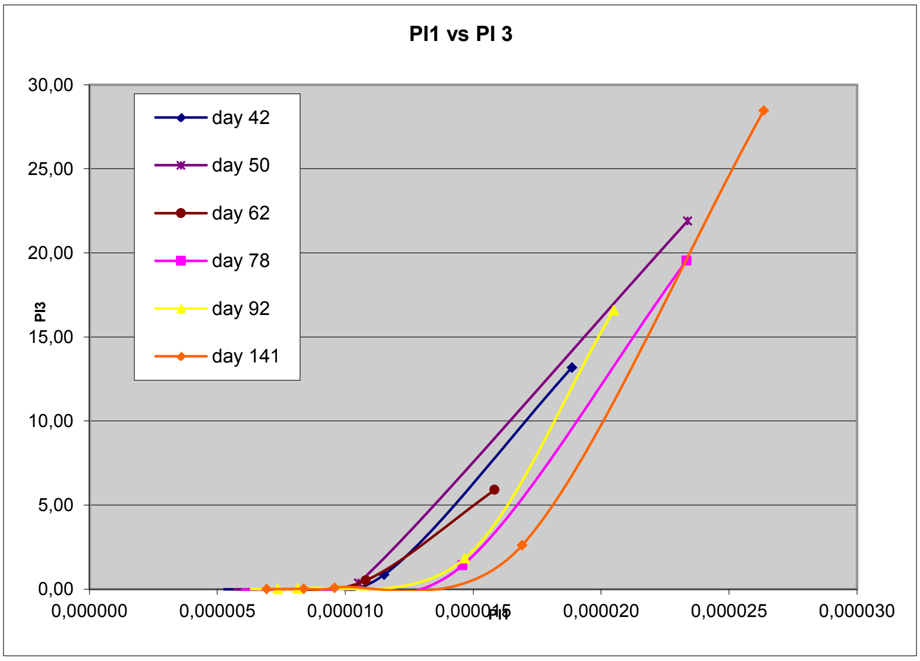

Figure 5 : Relationships of the first and the third dimensionless products

6.5 Fifth product vs. third product

A similar behaviour is observed for the relationship of the fifth and third dimensionless products. There is an asymptotic behaviour for the effective stress as a function of the excess density (see Figure 6).

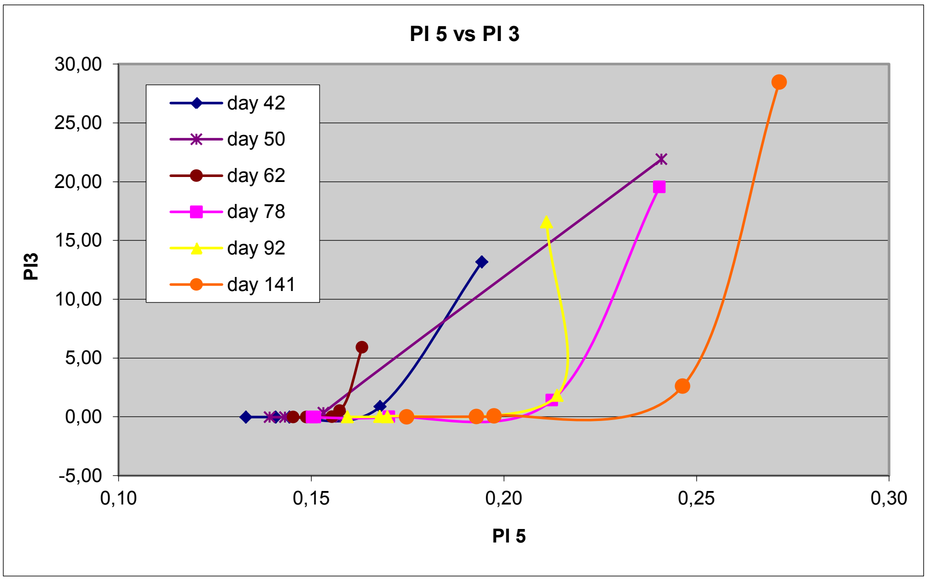

Figure 6 : Relationships of the fifth and the third dimensionless products

However, there are negative values of the effective stress close to the interface and for the initial days. These negative values are a result of the inconsistencies of the measured densities, negative stresses should not be present in saturated soils. Moreover, these inconsistencies are more evident for days 50, 62 and 92 with behaviour that does not follow the general trend.

6.6 First product vs. fourth product

Again, a similar behaviour is observed for the relationship of the first and fourth dimensionless products. There is an increment of the effective stress as a function of the excess density (see Figure 7).

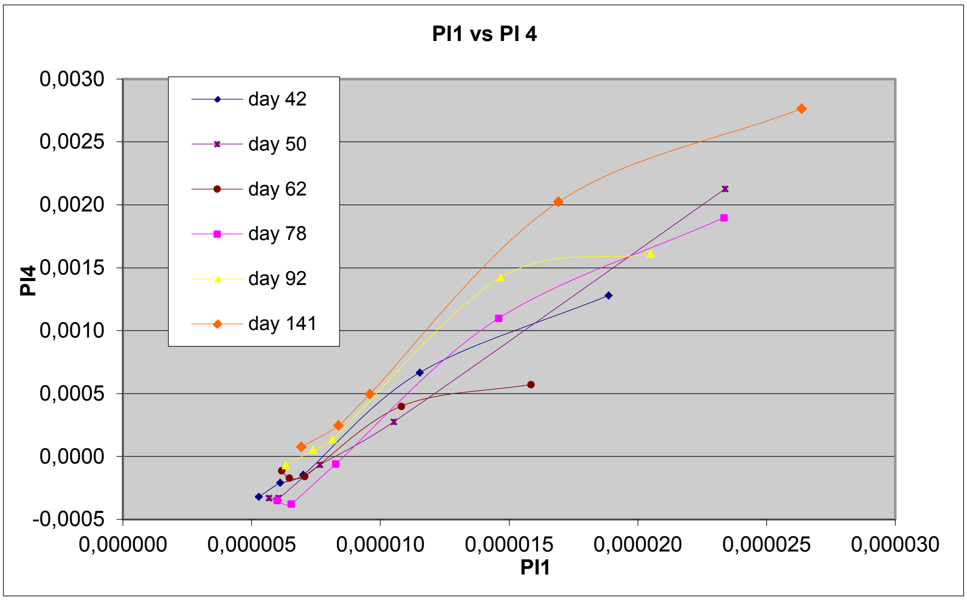

Figure 7 : Relationships of the first and the fourth dimensionless products

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6.7 Complementary products

As complementary products, one relationship is presented, following the criteria that the effective stress is a function of the density and other parameters. The relationship used for this purpose is fifth product vs. fourth product. This relationship is complementary to the previous ones, in this case the settling velocity is considered.

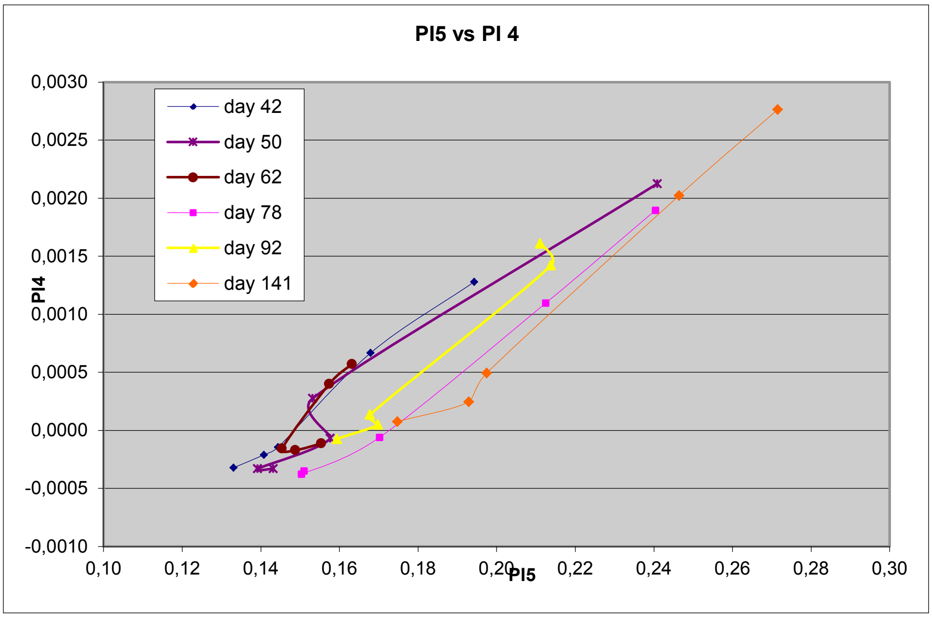

Figure 8 : Relationships of the fifth and the fourth dimensionless products

7 Conclusions and recommendations

7.1 Conclusions

After the application of the dimensional analysis technique, the following conclusions can be drawn:

- The measurements of the different parameters started 6 days after the release of the sediments into the tank. This is a limiting factor, because there is no data that corresponds to the initial stage of the sedimentation. Hence, it was not possible to determine the relationships (dimensionless products) for lower densities.

- The pore pressure was measured only at six points. Considering that the density was not measured at the bottom, the number of points that can be used in the calculation of the effective stress is reduced to five. It is advised to increase the number of points where the pore pressure is measured in order to obtain more complete dimensionless products that include more points in the vertical coordinate.

- The effective stresses that corresponds to days 42, 50, 62 and 78 present negative values at depths close to the interface. This can be explained by the densities used for the calculation of the total stress. The sampling points close to the bottom are more consolidated, they present higher values of density. On the contrary, the samples taken close to the interface are not fully consolidated, presenting a high uncertainty in the measured values of the density, leading to the calculation of negative values for the total stress. This observation is confirmed with the fact that negative values of the total stress disappear with along the time, due to the consolidation process. Hence, densities increase and the uncertainty is reduced, leading to the calculation of positive values for the total stress.

- The dimensional analysis has shown that the measured values of the density and pore pressure for days 42, 78 and 141 follow a physical behaviour, where the density, the effective stress increase with the depth and along the time. The effective stress will increase up to certain limit following an asymptotic behaviour close to the bottom, this behaviour was already observed by Toorman and Huysentruyt (1997).

7.2 Recommendations

The following recommendations are suggested for further research of the Sediment Test Tank experiment:

- The measured densities should be verified in order to filter out possible external influences, especially for the first days of the release of sediments.

- It is advised to install more measuring points for the pore pressure, in this way, it would be possible to have more points to build up the dimensionless products.

- It is advised as well to perform further filtering process in order to correct possible inconsistencies with the measured pore pressures. A common technique is the use of interpolations to correct possible inconsistencies in the measurements.

- Much of the inconsistencies with the measurement of the densities are related to samples with a low level of consolidation (low densities), it is advised to perform measurements with other devices in order to discard possible error measurements and reduce the uncertainty.

7.3 Proposed further tasks

The following tasks are proposed in order to obtain more complete relationships. This aims to enhance the dimensionless products with data measured after day 141. Hence, it would be possible to determine empirical relationships for the consolidation phase in agreement with the theory proposed by Prof. Erik Toorman. The following tasks are proposed:

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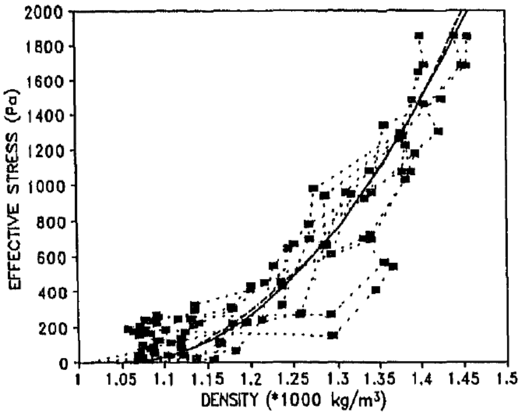
- Adjustment of the measured densities in order to filter-out the possible external influences (e.g. days 50, 62 and 92). Then inconsistencies will disappear and it would be possible to obtain relationships close to the observations reported by Toorman and Huysentruyt (1997). See figure below:



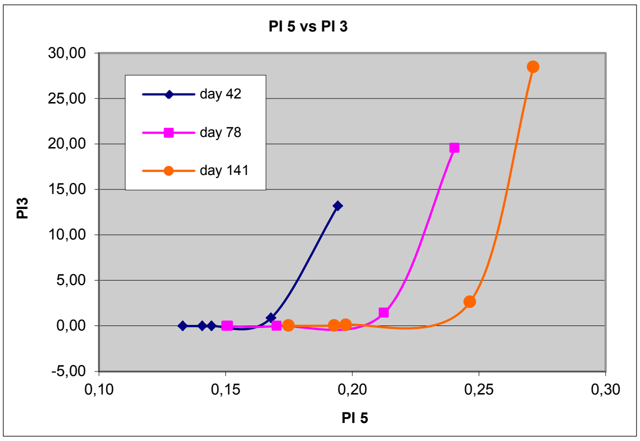


Figure 9 : Comparison of the reported relationship of effective stress and density of Toorman and Huysentruyt (1997) and the data measured from the STT experiment

- Adjustment of the measured densities in order to filter-out the negative values of the effective stresses. Then, positive values for the effective stresses. As a result, it would be possible to obtain logarithmic graphs and empirical relationships for the consolidation phase. The following figure depicts the dimensionless products (fourth vs. third) for day 141, with only positive values of the effective stress. This curve could be enhanced with the measurements obtained after this day.

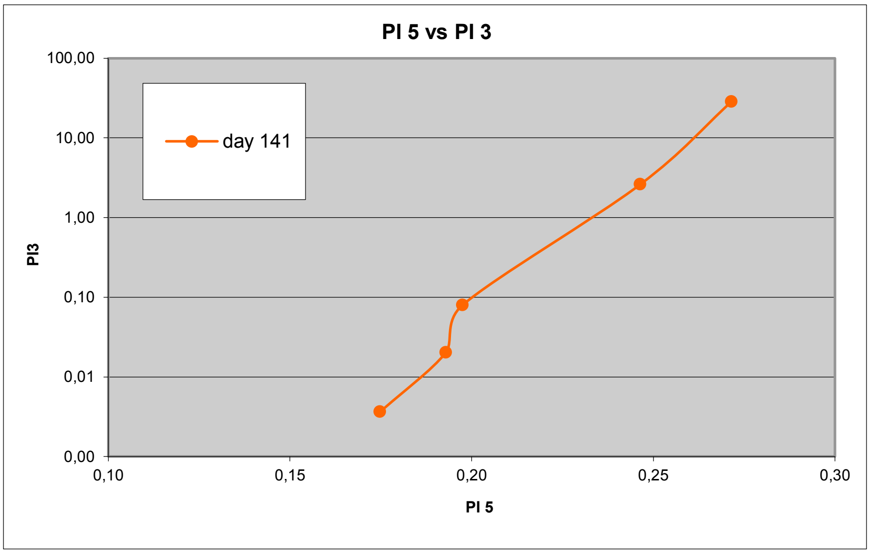


Figure 10 : Semi-logarithmic graph of the relationship of the dimensionless product (fifth vs. third) for day 142

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