# ON SINKAGE AND TRIM OF VESSELS NAVIGATING ABOVE A MUD LAYER

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## ABSTRACT

The paper presents a theoretical extension of a semi-empirical method for the determination of squat of ships in shallow water based on a one-dimensional theory, in which the solid bottom is replaced by a higher density fluid layer. It can be shown that, for the evaluation of the effect of the mud layer, a difference has to be made between three ranges of the ship's speed, separated by two critical values. The first critical value is approximately equal to the maximum velocity of propagation of internal waves at the interface, while the second critical value depends on the blockage factor and on the lower fluid density.

Numerical results of theoretical calculations are compared with experimental data of tests carried out at the Hydraulic Research Laboratory in Antwerp-Borgerhout with self-propelled ship models in restricted waters above a solid bottom and above a simulated mud layer.

## INTRODUCTION

Safe navigation in shallow waters (e.g. approach channels to harbours) requires a minimum water depth or a minimum keel clearance. If sediments are deposited in the navigation area considered, however, water and solid bottom are separated by a mud layer, so that the question arises which depth and keel clearance conditions have to be fulfilled. In those cases, it is necessary to introduce terms as "nautical depth" and "nautical bottom": the latter can be defined as a horizontal plane with particular characteristics, situated between the top of the mud and the solid bottom, above which a ship can still navigate and manoeuvre in a safe way.

The knowledge of the physical characteristics which are typical for this nautical bottom is very important for the optimization of maintenance dredging work in muddy canals and harbours. For this reason, a study program on this subject was proposed by the "Dienst der Kust" (Coastal Department) of the Belgian Ministry of Public Works, jointly with Haecon nv and Decloedt nv. This study appeared to be necessary, as data concerning the behaviour of ships with restricted keel clearance above mud are hard to find in literature: only in the Netherlands, both full scale (Rotterdam) and model (MARIN, Wageningen) experiments have been carried out on oil tankers, (see [1],[2]).

The "Waterbouwkundig Laboratorium" (Hydraulic Research Laboratory) in Antwerp-Borgerhout, scientifically supported by the "Dienst voor Scheepsbouwkunde" (Office of Naval Architecture) of the State University of Ghent, was involved in this study, as it was expected that model tests would take an important place in the evaluation of the total effect of the presence of a mud layer on a ship's performance.

In this paper, one particular aspect of the study is presented : the influence of the presence of a mud layer on the vertical displacement of a ship. This aspect might be of interest, not only because of the importance of squat and trim in determining the allowable keel clearance, but also because theoretical calculations of sinkage reveal some characteristics of the motions of the interface. These theoretical calculations can be considered as an extension of a semi-empirical method for calculating squat and trim of full ships in shallow water. This method has been developed by Dand and Ferguson, [3], and is in fact only semi-empirical if it is used for calculating vertical displacements of ships navigating in shallow water of a considerable width, as it is based on a one-dimensional theory: in narrow channels, however, no empirical assumptions are needed.

Numerical results of theoretical calculations are compared with experimental values obtained from model tests, carried out in the Hydraulic Research Laboratory.

## MODEL EXPERIMENTS

### Introduction

Handling of problems concerning a ship's behaviour in restricted waters is of increasing importance for the Hydraulic Research Laboratory. This is the reason why the construction of a 70 x 7 x 0.6 m shallow-water tank for ship model experiments, equipped with a towing carriage and a planar motion mechanism, is planned.

Nevertheless, it has been decided not to wait for the realization of this equipment to carry out an experimental program for studying the influence of the presence of a mud layer on a ship's performance in a basin with more restricted dimensions, 32.00 x 2.25 x 0.3 m.

Although in general a length restriction causes difficulties in obtaining a steady-state condition for the ship model, the consequences for the test program considered were not too important, because only the lower speed range was of interest. On the other hand, a width restriction causes blockage effects, but the latter can also be expected in reality, as most problems with mud layers occur in canals or dredged channels.

Moreover, the main purpose of the test program was not to obtain quantitative information, but to select a suitable material to simulate mud on model scale, and to obtain a better insight into the physical causes of the changements in a ship's behaviour.

## Experimental set-up

The tests were carried out in a small basin with dimensions 32.00 x 2.25 x 0.30 m. The basin floor was levelled with an accuracy of ±1 mm; floor and walls were protected against penetration of the mud simulating material.

Two ship models, scale 1:70, were selected : a third-generation container carrier (model dimensions : Lpp = 3.53 m ; B = 0.46 m ; T = 0.17 m ; Cb = 0.68) and a product tanker (model dimensions : Lpp = 3.81 m ; B = 0.59 m ; T = 0.16 m ; Cb = 0.80) . The first-mentioned was only used for tests above a solid bottom, while with the other ship model experiments above a layer of mud simulating material were carried out as well. The models, equipped with rudder and propeller, were forced to follow the centreline of the basin by means of a guiding beam, but were able to move freely in vertical sense (see fig. 1).

A wireless communication system between the ship model and a personal computer was developed for control of propulsion and rudder, and for acquisition of the measured data. The latter were transmitted in a number of equidistant points of the guiding beam, separated 0.25 m from each other, and consisted of :

- time between two measuring points;
- vertical distance to guiding beam in two measuring posts (MA and MF);
- lateral force in the two measuring posts.

In one particular point of the basin the interface motions were registrated by means of a profile following device.

## Mud simulating material.

Due to the particular rheological properties of mud, it is extremely difficult to find a material able to simulate it in all its aspects for model tests.

It is known that, for ship model tests carried out in water, it is impossible to follow both Froude and Reynolds conditions; the interpretation of resistance tests therefore implies an extrapolation technique consisting in separating the total resistance in two parts, and supposing that the friction part depends on the Reynolds number, while the residual part is independent of viscosity and is a function of the Froude number only.

When tests have to be carried out with ship models navigating above a mud simulating material, the interpretation difficulties mentioned are still increased. The Froude condition can be fulfilled by choosing a mud simulating material with the same density, but it will be extremely difficult to find a material with which both Froude and Reynolds laws are followed. Moreover, the behaviour of the mud layer is influenced by other rheological characteristics, such as shear stress and yield stress. Another complication is caused by the fact that the physical characteristics mentioned are variable with depth and with time.

Several materials which might be accepted for mud simulation have been studied by the

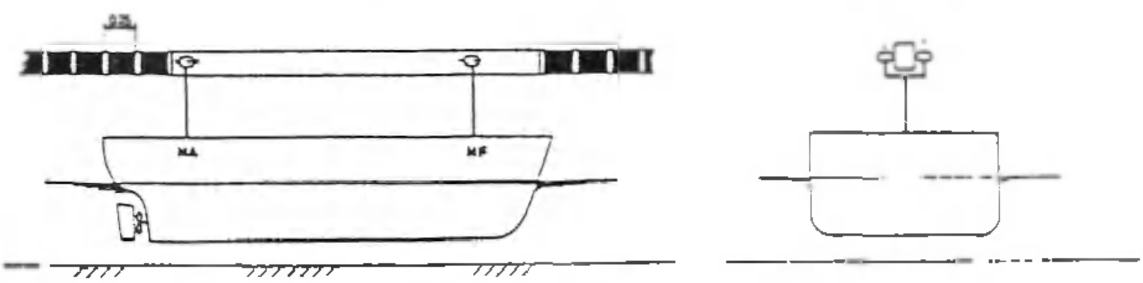



Fig. 1. Experimental set-up (schematic representation)

Hydraulic Research Laboratory and by Haecon nv :

- natural mud, the rheological characteristics of which are scaled by means of chemical additives;
- artificially composed mud;
- organic liquids.

Although the latter are not able to represent several important characteristics, their use offers some advantages :

- their characteristics do not change with time, which is an important advantage in this early investigation stage;
- the validity of theoretical developments, based on the behaviour of a system consisting of two ideal fluid layers, can be checked, which permits the reference of the behaviour of a ship navigating above a real mud layer to an "ideal" situation.

A mixture of trichlorethane and petrol was selected for simulating the mud. The density of the fluid can be adjusted by changing the amount of petrol.

This material offers the following advantages :

- solvability in water is zero;
- although the rheological properties are not scaled exactly, the differences are expected to be acceptable.

Test program.

The test program consisted of acceleration tests, steady-state tests (with constant speed), deceleration tests and rudder angle tests.

These experiments were carried out at several values of keel clearance, varying from +0.2 T to +0.02 T for tests above solid bottom, and from +0.2 T to -0.06 T for tests with a two-layer system.

It is not the purpose of this article to give a complete review of the results of this test program. A selection of experimental results will only be given to illustrate or to confirm results of theoretical calculations.

## THEORETICAL BACKGROUND

Conventions (see fig. 2)

The ship is moving forward with speed U in a canal of width w. The solid bottom of the canal is covered with a higher density fluid (mud) layer of thickness $\delta$; the water depth, referred to the top of this layer is denoted $h_1$. The densities of the upper and lower fluid layers are presented by $\rho_1$ and $\rho_2$, respectively.

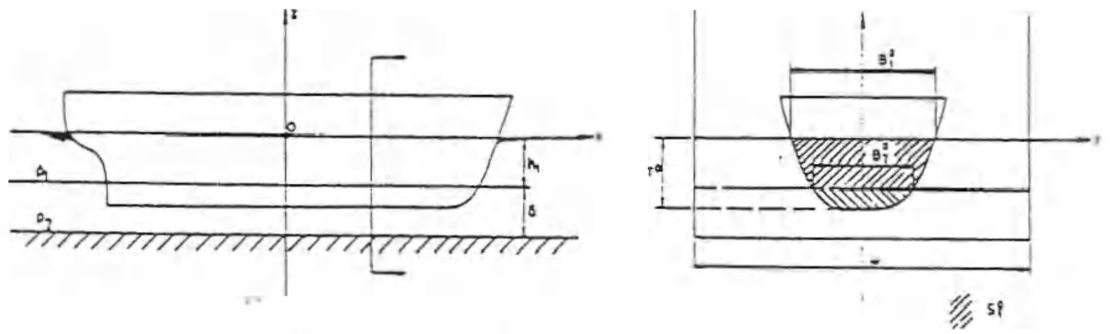



Fig. 2a. Geometry in initial position (U = 0).

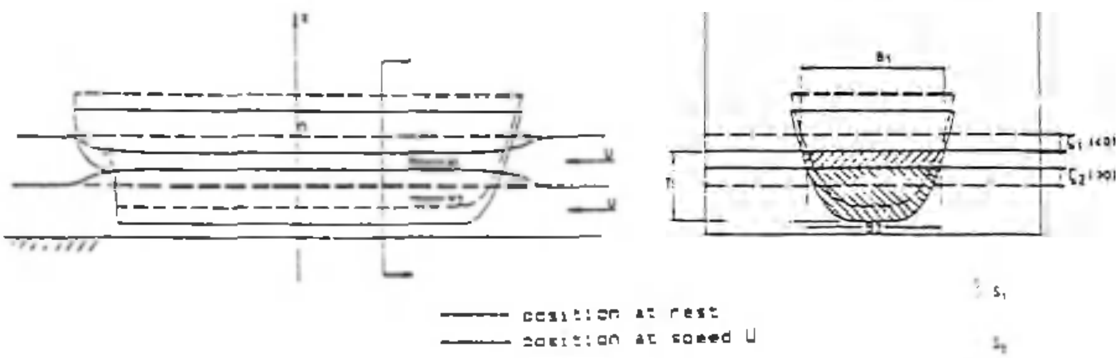



Fig. 2b. Geometry while navigating with speed U.

A carthesian right-handed co-ordinate system Oxyz is moving with the ship, so that the origin O is situated on the hull centerline in the waterplane : the Ox-axis in the longitudinal direction, pointing to the bow; the Oz-axis vertically upward; the Oy-axis laterally, pointing to port.

It is assumed that the disturbance due to the ship motion is constant over a given cross-section, so that perturbations in the y- and z-directions are neglected. This means that the following fluid velocities and surface positions are variables of the longitudinal co-ordinate x :

- velocity of the upper fluid (water) : $u_1(x)$
- velocity of the lower fluid (mud) : $u_2(x)$
- free surface position : $\zeta_1(x)$
- interface position : $\zeta_2(x)$

The velocities $u_1$ and $u_2$ are referred to the moving co-ordinate system Oxyz, so that $u_1<0$ and $u_2<0$.

Fluid layer velocities.

As the reference frame is moving with the ship, the problem is reduced to one of steady flow, in which the ship's position is fixed while the two fluid layers are moving with a velocity -U.

Taking account of the simplifications mentioned above, continuity requires that :

$$-U\,w\,h_1 = u_1(x)\Big[w\,[h_1 + \zeta_1(x) - \zeta_2(x)] - S_1(x)\Big] \qquad (1)$$

$$-U\,w\,\delta = u_2(x)\Big[w\,[\delta + \zeta_2(x)] - S_2(x)\Big] \qquad (2)$$

where $S_1(x)$ represents the part of the sectional area S(x) between the free surface and the interface, and $S_2(x)$ the part under the interface. When the underkeel clearance referred to the top of the "mud" layer is sufficiently large, $S_2$ equals zero.

On the free surface, application of Bernoulli's equation yields :

$$\frac{1}{2}U^2 = \frac{1}{2}u_1^2 + g\,\zeta_1 \qquad (3)$$

On the interface, the boundary condition is given by dynamic pressure matching :

$$\rho_2\left[\frac{1}{2}u_2^2 + g\,\zeta_2\right] - \rho_1\left[\frac{1}{2}u_1^2 + g\,\zeta_2\right] = \frac{1}{2}(\rho_2 - \rho_1)\,U^2 \qquad (4)$$

The following Froude numbers are now defined :

$$F_1^2 = \frac{U^2}{g\,h_1} \qquad (5)$$

$$F_2^2 = \frac{U^2}{g\,\delta\left[1 - \frac{\rho_1}{\rho_2}\right]} \qquad (6)$$

If $\zeta_2$ is eliminated from equations (2) and (4), the following expression is obtained :

$$f_2\left[-\frac{u_2}{U}\right] = 1 - \left[1 - m_2 + \frac{1}{2}F_1^2\left[1 - \frac{\rho_1}{\rho_2} + \frac{\rho_1}{\rho_2}\left[-\frac{u_2}{U}\right]^2\right]\right]\left[-\frac{u_2}{U}\right] + \frac{1}{2}F_2^2\left[-\frac{u_2}{U}\right]^3 = 0 \qquad (7)$$

Elimination of $\zeta_1$ from equations (1) and (3) yields :

$$1 - \left[1 - m_1 - \frac{\zeta_2}{h_1} + \frac{1}{2}F_1^2\right]\left[-\frac{u_1}{U}\right] + \frac{1}{2}F_1^2\left[-\frac{u_1}{U}\right]^3 = 0 \qquad (8)$$

or, taking account of (2) :

$$f_1\left[-\frac{u_1}{U}\right] = 1 - \left[1 - m_1 + \frac{1}{2}F_1^2\,\frac{1}{1 - \frac{\rho_1}{\rho_2}}\left[-\frac{u_1}{U}\right]^2\right]\left[-\frac{u_1}{U}\right] + \frac{1}{2}F_1^2\,\frac{1}{1 - \frac{\rho_1}{\rho_2}}\left[-\frac{u_1}{U}\right]^3 = 0 \qquad (9)$$

In these expressions, $m_1(x)$ and $m_2(x)$ are the local blockage factors of the upper and lower fluid layers, respectively :

$$m_1(x) = \frac{S_1(x)}{w\;h_1} \qquad (10)$$

$$m_2(x) = \frac{S_2(x)}{w\;\delta} \qquad (11)$$

Equations (7) and (9) provide a system of two nonlinear equations with two unknown variables, $-u_1/U$ and $-u_2/U$. In fact this system can only be solved by iteration, as the blockage factors $a_1(x)$ and $a_2(x)$ vary with the local sinkage and the local free surface and interface elevations.

## Vertical displacement.

Sinkage and trim can be calculated as follows. It can be shown that the buoyancy force per length unit in a section of the ship hull is given by :

$$F_z(x) = g\Big[\rho_1\ B_1\ [\zeta_1 - Z(x)] - (\rho_1 - \rho_2)\ (B_2\ \zeta_2 + S_2 - S_2^*)\Big] \qquad (12)$$

where $S_2^*$ denotes the part of the section area under the interface at rest.

The local sinkage $Z(x)$ is given by :

$$Z(x) = Zm + x\ \tau \qquad (13)$$

where $Zm$ and $\tau$ are sinkage midships and trim, respectively.

The total vertical force and the moment about the Oy-axis have to equal zero :

$$\int_{-\frac{1}{2}L}^{\frac{1}{2}L} F_z(x)\ dx = 0 \qquad (14)$$

$$\int_{-\frac{1}{2}L}^{\frac{1}{2}L} F_z(x)\ x\ dx = 0 \qquad (15)$$

Insertion of (12) and (13) yields :

$$\int_{-\frac{1}{2}L}^{\frac{1}{2}L}\Big[\rho_1\ B_1\ (\zeta_1 - Zm - \tau x) - (\rho_1 - \rho_2)(B_2\ \zeta_2 + S_2 - S_2^*)\Big]\ dx = 0 \qquad (16)$$

$$\int_{-\frac{1}{2}L}^{\frac{1}{2}L}\Big[\rho_1\ B_1\ (\zeta_1 - Zm - \tau x) - (\rho_1 - \rho_2)(B_2\ \zeta_2 + S_2 - S_2^*)\Big]\ x\,dx = 0 \qquad (17)$$

which leads to the following expressions for $Zm$ and $\tau$ :

$$Zm = \frac{Co\ A_2 - C_1\ A_1}{Ao\ A_2 - A_1^2} \qquad (18)$$

$$\tau = \frac{C_1\ Ao - Co\ A_1}{Ao\ A_2 - A_1^2} \qquad (19)$$

where

$$An = \int_{-\frac{1}{2}L}^{\frac{1}{2}L} B_1\ x^n\ dx \qquad (n=0,1,2) \qquad (20)$$

$$Cn = \int_{-\frac{1}{2}L}^{\frac{1}{2}L} B_1\ \rho_1\ x^n\ dx + \left[1 - \frac{\rho_2}{\rho_1}\right]\int_{-\frac{1}{2}L}^{\frac{1}{2}L} (B_2\ \zeta_2 + S_2 - S_2^*)\ x^n\ dx \qquad (n=0,1) \qquad (21)$$

Note : if the centre of gravity of the water-plane area is chosen to be the origin O, expressions (18) and (19) are reduced to

$$Zm = \frac{Co}{Ao} = \frac{\displaystyle\int_{-\frac{1}{2}L}^{\frac{1}{2}L} B_1\ \zeta_1\ dx + \left[1 - \frac{\rho_2}{\rho_1}\right]\int_{-\frac{1}{2}L}^{\frac{1}{2}L} (B_2\ \zeta_2 + S_2 - S_2^*)\ dx}{\displaystyle\int_{-\frac{1}{2}L}^{\frac{1}{2}L} B_1\ dx} \qquad (22)$$

$$\tau = \frac{C_1}{A_2}$$

$$= \ldots$$

$$= \frac{\int_{-½L}^{½L} B_1 \zeta_1 \, x dx + \left[1 - \frac{\rho_1}{\rho_2}\right] \int_{-½L}^{½L} (B_2 \zeta_2 + S_2 - S_2^*) x dx}{\int_{-½L}^{½L} B_1 x^2 \, dx} \qquad (23)$$

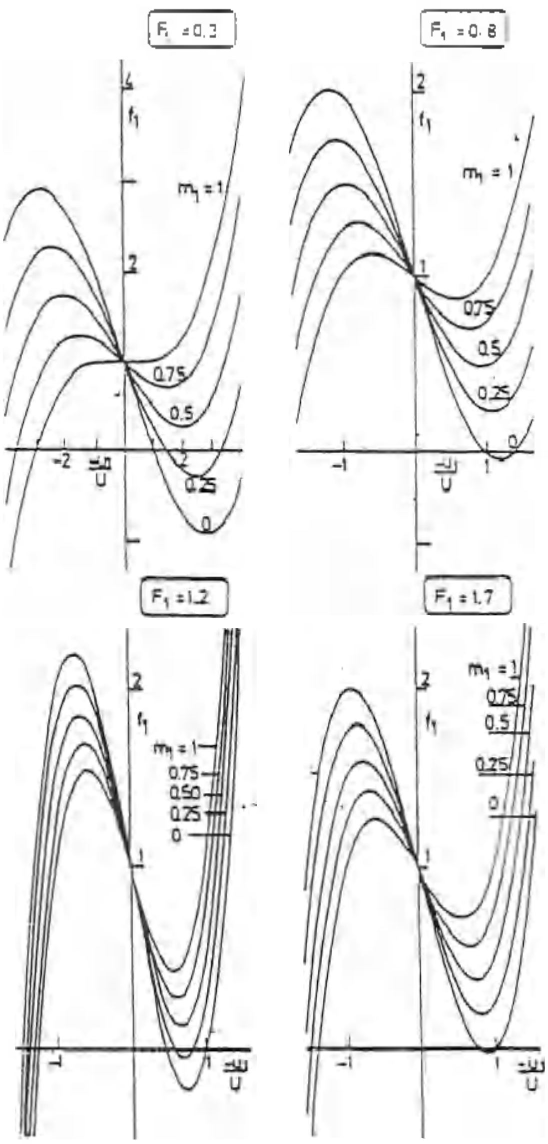



Fig. 3. Solid bottom : function $f_1(-u_1/U)$ for several values of Froude number $F_1$ and blockage factor $m_1$.

## SHIP NAVIGATING ABOVE A SOLID BOTTOM

### Theoretical calculations.

If the lower fluid layer is not present, expression (8) takes the following form :

$$\frac{1}{2} F_1^2 \left[-\frac{u_1}{U}\right]^3 - \left[1 - m_1 + \frac{1}{2} F_1^2\right] \left[-\frac{u_1}{U}\right] + 1 = 0 \qquad (24)$$

The number of positive roots of third-order polynomial equation (24) can be 2, 1 or 0, depending on the values of $F_1$ and $m_1$ (see fig. 3). According to Constantine, [4], no real solution can be found in a critical velocity range. For subcritical values of $F_1$, the smaller positive root gives the solution for $(-u_1/U)$, while for supercritical values, $(-u_1/U)$ is given by the larger one. As it has been shown by Schijf, [5], it is theoretically impossible for a self-propelled ship to exceed the subcritical velocity range: therefore, the critical and supercritical ranges will not be considered here.

Expressions (18) and (19) are still valid, but the second term in the expression for Cn, (21), disappears. If the centre of gravity of the waterplane area is chosen to be the origin O, sinkage and trim can be expressed as follows :

$$Zm = \frac{\int_{-½L}^{½L} B_1 \zeta_1 \, dx}{\int_{-½L}^{½L} B_1 \, dx} \qquad (25)$$

$$\tau = \frac{\int_{-½L}^{½L} B_1 \zeta_1 \, dx}{\int_{-½L}^{½L} B_1 x^2 \, dx} \qquad (26)$$

These expressions can also be found in a paper by Dand and Ferguson, [3].

Theoretically, the vertical displacement of the ship can only be calculated by iteration as the blockage factor $m_1$ is a function of the local sinkage of the ship $Z(x)$ and the free surface elevation $\zeta_1(x)$ :

$$m_1 = m_1{}' + \frac{S_1 - S_1{}'}{w\ h_1} \qquad (27)$$

where $m_1{}'$ represents the local blockage factor at rest, $S_1{}'$ the sectional area for initial draught $T = T'$, and $S_1$ the sectional area for draught $T = T' + Z - \zeta_1$. If variations of the local beam $B_1$ with draught are not too important, (27) can be written approximately as :

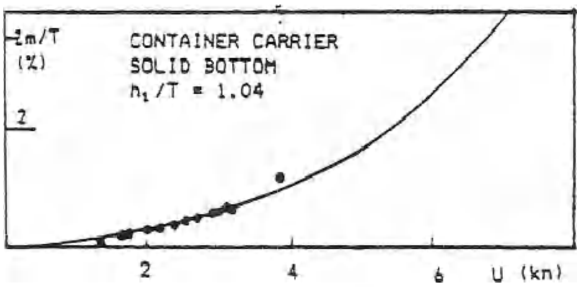



Fig. 4. Container carrier navigating above solid bottom - keel clearance 4 % of draught : trim angle.
Theoretical and experimental results.

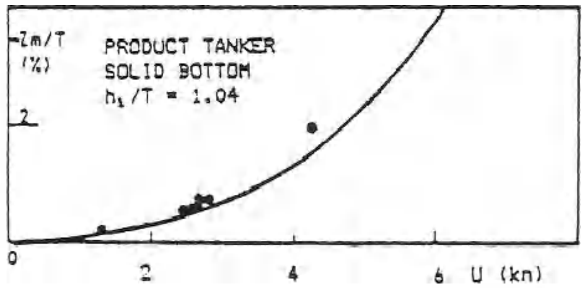



Fig. 5. Product tanker navigating above solid bottom - keel clearance 4 % of draught : mean sinkage.
Theoretical and experimental results.

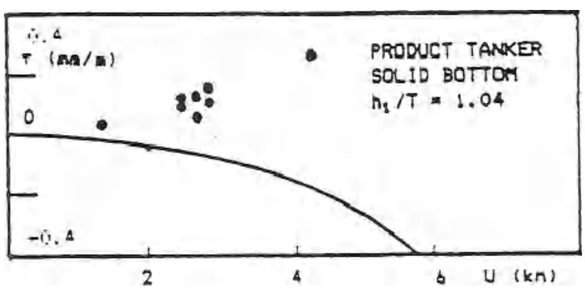



Fig. 6. Product tanker navigating above solid bottom - keel clearance 4 % of draught : trim angle.
Theoretical and experimental results.

$$m_1 \cong m_1{}' + \frac{B_1\ (\zeta_1 - Z)}{w\ h_1} \qquad (28)$$

In most cases, the second term of the right-hand side of (28) can be neglected :

$$m_1 \cong m_1{}' \qquad (29)$$

so that in practice there is no need for an iteration method for calculating sinkage and trim.

Experimental results.

An extensive experimental program has been carried out with the self-propelled ship models navigating above a solid bottom with several values of underkeel clearance. The results of these tests concerning vertical ship model displacements, together with the theoretical values, are shown in figs. 4, 5 and 6.

Mean sinkage and trim seem to be underestimated slightly by theory: this fact can be explained by the influence of self-propulsion. This effect has been treated by Dand and Ferguson, [3].

## SHIP NAVIGATING ABOVE A MUD LAYER

Theoretical developments.

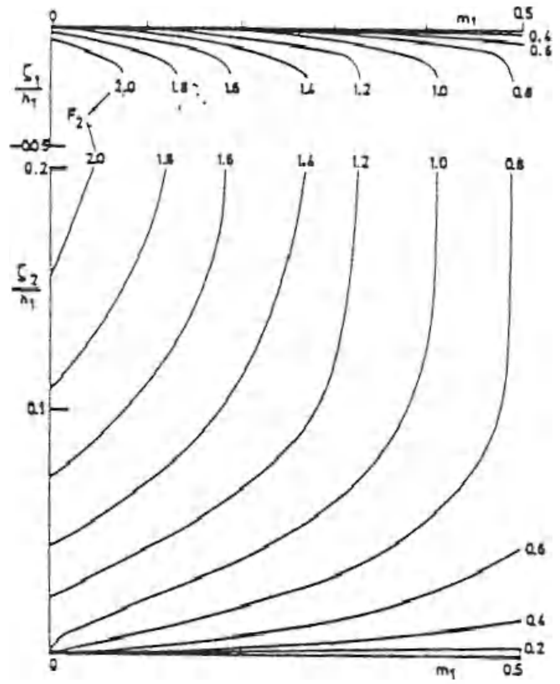
If the underkeel clearance of the ship referred to the interface is sufficiently large, the blockage factor $m_2$ for the lower fluid layer equals zero, which causes a slight simplification of expression (7).

The system provided by equations (7) and (9) can theoretically deliver, as a maximum, four combinations of real, positive values [ $(-u_1/U)$ , $(-u_2/U)$ ]. As only subcritical Froude numbers $F_1$ are considered here, only the smaller positive root of (7) will be taken into account; as a result, the number of real, positive solutions of the system can be 0, 1 or 2.

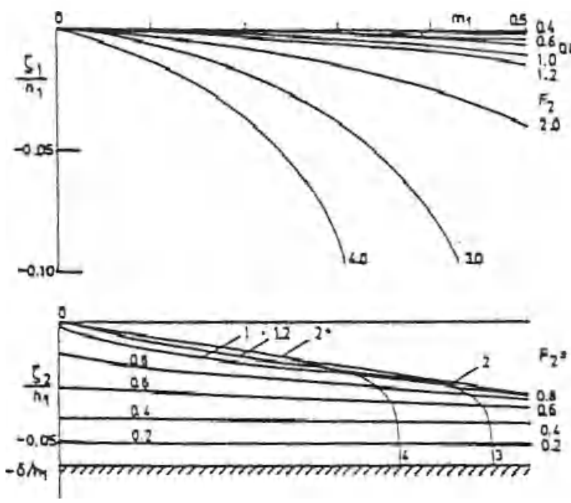
As an example, free surface and interface elevations $\zeta_1$ and $\zeta_2$ are shown in fig. 7 in function of $m_1$ and $F_1$, for one particular layer configuration $(\delta/h_1\ ,\ \rho_2/\rho_1)$. It appears that :

- normally, two situations are possible : one resulting into an elevation of the interface ($\zeta_2 > 0$ ; $-u_2/U < 1$), another into a sinkage ($\zeta_2 < 0$ ; $-u_2/U > 1$);

- for larger Froude numbers $F_1$ and/or larger blockage factors $m_1$, the situation resulting into an interface elevation is not possible:




(a) interface elevation




(b) interface sinkage

Fig. 7. Two-layer system : $\rho_2/\rho_1$ = 1.14 ; $\delta/h_1$ = 0.06. Possible free surface and interface motions in function of blockage factor $m_1$ and Froude number $F_2$.

- as the initial condition ($m_1$ = 0 ; $\zeta_2$ = 0) is situated on only one of the curves in fig. 7, one can expect that small Froude numbers $F_2$ will cause an elevation, while large values will result into a sinkage of the interface.

Obviously, two critical speed values, and three speed ranges can be defined :

- at low speeds, both elevation and sinkage of the interface are possible, but the first solution is expected to be the most "natural" one;
- at higher speeds, both solutions are possible as well, but one can expect an interface sinkage;
- in the highest speed range, only an interface sinkage can occur.

In fig. 8, where it is shown that for all values of $(-u_1/U) > 1$, one or two real, positive roots of equation (7) can be found, so

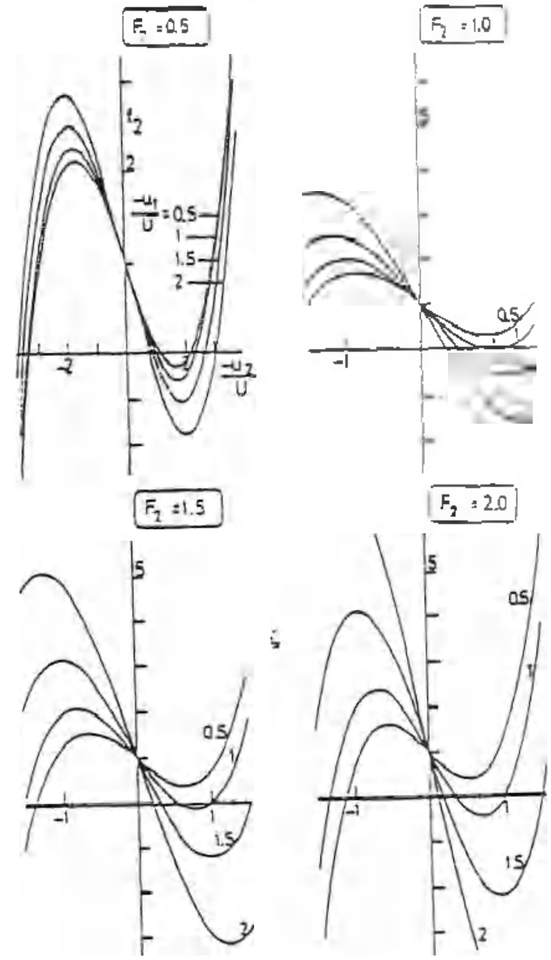



Fig. 8. Two-layer system : $\rho_2/\rho_1$ = 1.14 : $\delta/h_1$ = 0.06. Function $f_2(-u_2/U)$ for several values of Froude number $F_2$ and relative velocity factor $-u_1/U$.

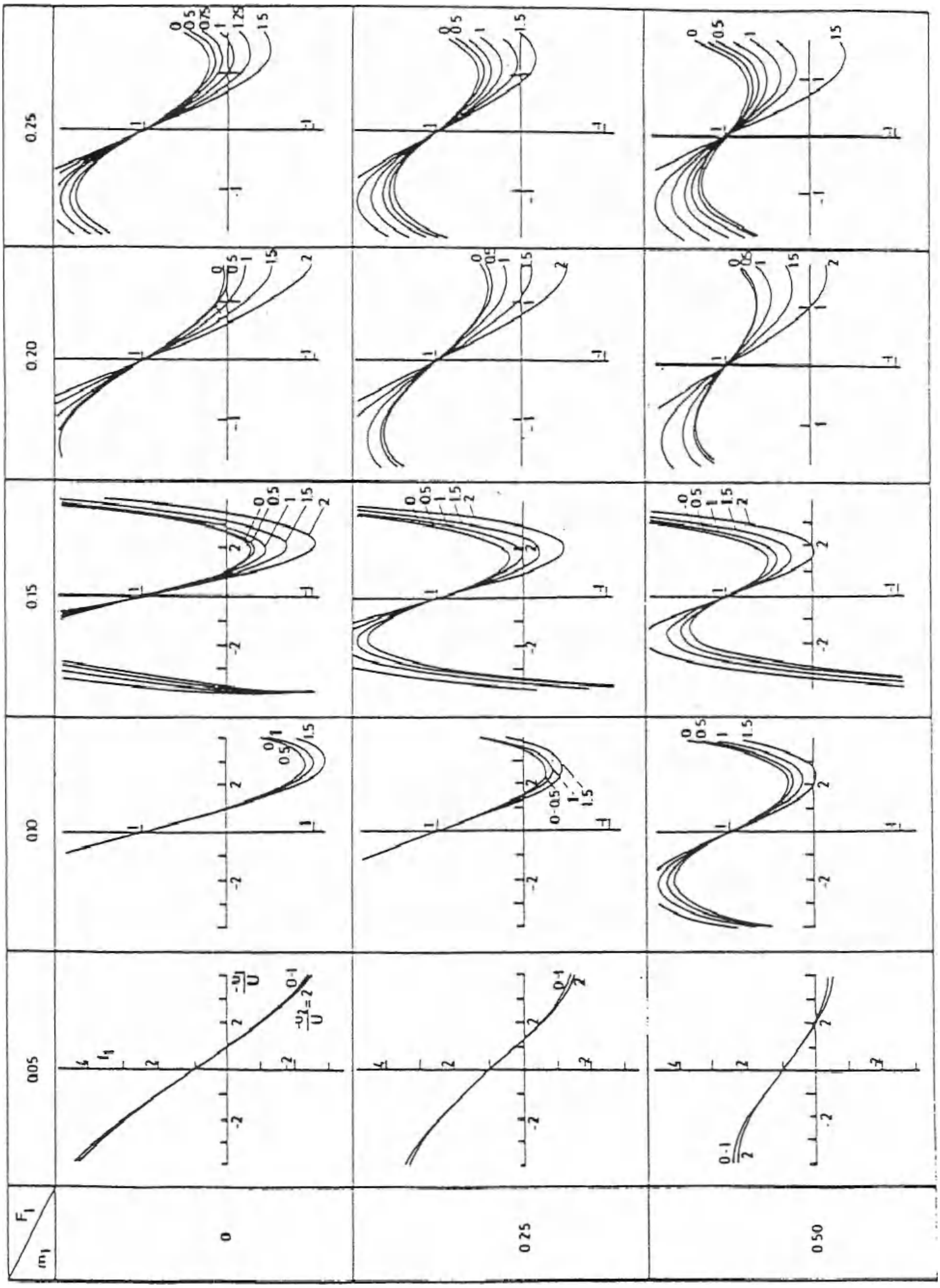



Fig. 9. Two-layer system : $\rho_2/\rho_1$ = 1.14 ; $\delta/h_1$ = 0.06.
Function $f_1(-u_1/U)$ for several values of Froude number $F_1$,
lower fluid layer velocity factor $-u_2/U$ and blockage factor $m_1$.

4.158 that the number of solutions will always be 1 or 2.

On the other hand, fig. 9 shows that the number of positive, real roots of equation (9) depends on $(-u_1/U)$ and $m_1$. It seems that for lower values of $(-u_1/U)$, such roots cannot be found if the values of $F_1$ or $m_1$ are too high; this fact explains the existance of the second critical speed value.

Observation of the interface during experiments

With the self-propelled 1/70 scale model of a product carrier, model tests have been carried out above a fluid layer with density 1140 kg/m3 and thickness 7 % of the draught. During these tests, the underkeel clearance referred to the interface was varied from +20 % to -6 %.

At low speed, $F_1 < 1$, a small interface sinkage could be observed near the forebody. Under the parallel middlebody, this sinkage gradually disappeared and changed into an interface elevation. The initial interface sinkage cannot be predicted by theory, but this is probably caused by the simplifying assumptions made during the theoretical developments. In fact, the flow around the ship hull is not one-, but three-dimensional; especially near the ship entrance, vertical and lateral velocities cannot be neglected. On the other hand, the effect of these simplifications on ship sinkage and trim is very small, especially at the very low speeds considered.

When the ship's speed exceeded the first critical value, $F_1 = 1$, an interface sinkage was observed under the entrance, but at some section, this sinkage suddenly changed into an elevation. This phenomenon showed much resemblance with a hydraulic jump in channels, especially because the profile of the interface jump described here also developed undulations, which also occur in channel flows at moderate Froude numbers ($1 < F < \sqrt{3}$), see Wehausen, [6]. The section at which the interface jump occurred, moved towards the stern with increasing speed. The angle between the wave front and the canal centerline was approximately 90°, which emphasizes the one-dimensional character of the flow in this speed range.

With increasing speed, the third critical value (which depends on the blockage factor) was exceeded for the parallel middlebody, so that the interface jump could only occur under the ship's afterbody, or, finally, behind the stern. The angle between the wave front and the canal centerline increased from 90° to approximately 135°, and the interface elevation attained values which exceeded the lower fluid layer thickness several times.

The profile of the interface for several speeds is shown in fig. 10, where the experimental results can be compared with the theoretically calculated curves of fig. 7.

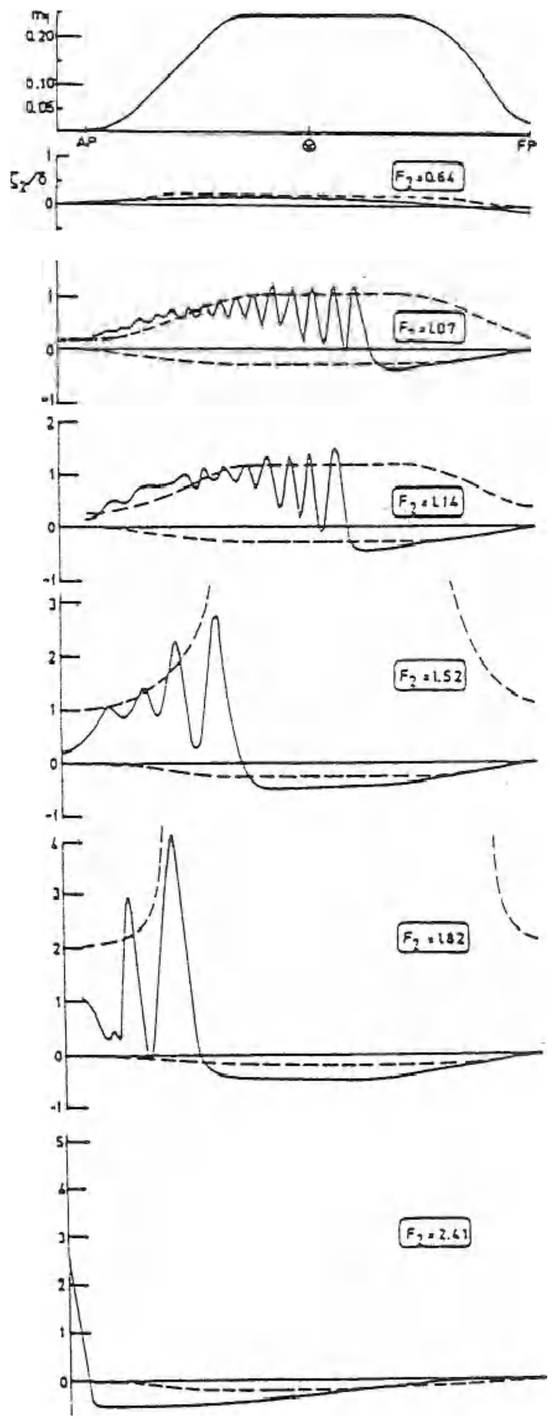



Fig. 10. Product tanker navigating above a two-layer system : $\rho_2/\rho_1 = 1.14$ ; $\delta/h_1 = 0.06$. Interface motions for several Froude numbers $F_1$ : theoretical predictions and experimental observations.

## Vertical displacements

Sinkage and trim can only be calculated if the position of the interface jump is known. If it is assumed that, for the speed range between the second and third critical speed values, this position moves gradually with speed from the fore end to the aft end of the parallel middlebody, fig. 11 is obtained.

The effect of the presence of the lower fluid layer on sinkage and trim depends on the ship's speed :

- for a ship moving at low speeds ($F_I < 1$), the layer causes a very slight increase of mean sinkage;
- for a ship moving at a speed higher than the second critical value, a sinkage decrease can be observed;

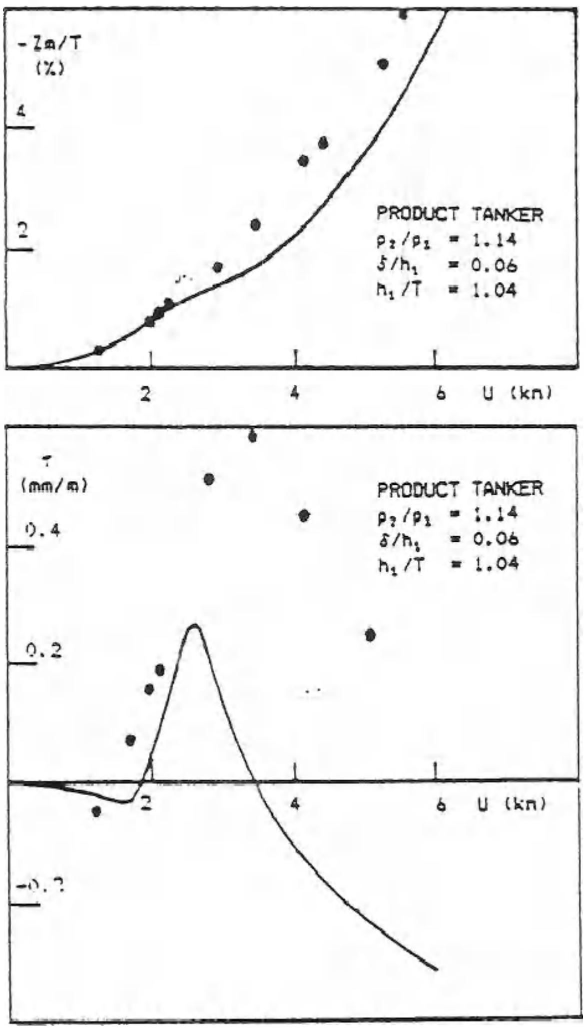



Fig. 11. Product tanker navigating above a two-layer system : $\rho_2/\rho_1 = 1.14$ ; $\delta/h_1 = 0.06$ - keel clearance 4 % of draught. Mean sinkage and trim : theoretical and experimental values

- for values of speed in the second range, the effects of the layer will result into an increase of the trim angle.

Fig. 11 also shows that the sign of the trim angles can be reversed if a solid bottom is replaced by a muddy one.

The agreement between experimental and theoretical values seems to be good for speed values in the first and second speed ranges. For higher speeds, the character of the flow can probably not be described in an effective way by a one-dimensional approximation, and it can be expected that the influence of the propulsion cannon be neglected in this speed range.

## Existance of the second speed range.

The second speed range only exists if the second critical speed value is larger than 1, i.e. the first critical speed.

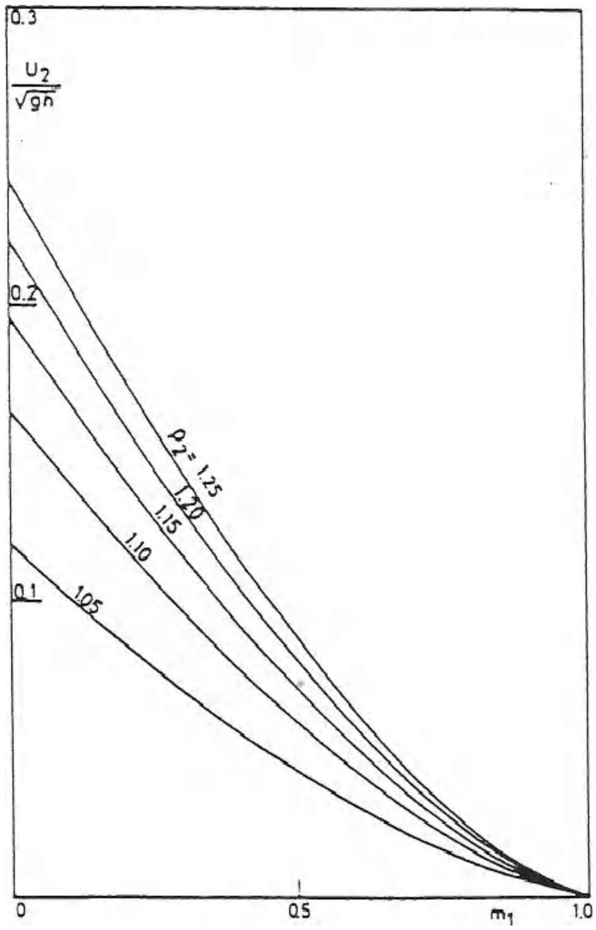



Fig. 12. Second critical speed value.

As can be seen in fig. 9, the dependence of the function $f_1(-u_1/U)$ on the value of the parameter $(-u_1/U)$ decreases with decreasing values of the latter, so that an approximative expression for this speed value can be obtained if $(-u_1/U)$ is supposed to be zero :

$$F^2_{crit} = \frac{U_1^2}{gh_1} \approx \frac{8}{27} (1 - m_1)^3 \left(1 - \frac{\rho_1}{\rho_2}\right) \qquad (30)$$

The value of this critical Froude number is shown in fig. 12.

Hence, the criterium for the existance of the second speed range, and, therefore, the presence of an interface jump can be expressed as follows (see also fig. 13) :

$$\frac{\delta}{h_1} < \left(\frac{\delta}{h_1}\right)_{crit} = \frac{8}{27} (1 - m_1)^3 \qquad (31)$$

## SHIP NAVIGATION IN A MUD LAYER (NEGATIVE KEEL CLEARANCE)

Although navigation of ships with negative keel clearance referred to the interface are beyond the scope of this paper, the theoretical developments described in this article provide a base for handling problems of that kind. However, calculations will be far more complicated because of the effect of interface elevation and local ship sinkage on the blockage factor for each layer, especially the lower one. A correct evaluation will require the introduction of Bonjean curves in the calculation scheme.

Experiments have shown that, concerning the behaviour of the interface and the effect on the performance of the ship model, no fundamental difference can be observed between tests carried out with positive or negative keel clearance. This is especially the case for the speed range between the first and the second critical speed : because of the interface sinkkage in the forebody and the elevation in the afterbody of the ship, there is an important range of keel clearances for which the ship is navigating partly above, and partly in the lower fluid layer.

When the whole ship's body is interface-piercing, for some speeds a bow wave can be observed in the interface. The height of this wave (about 2 mm) is very restricted compared with the elevations and sinkages due to "hydraulic" action. This phenomenon shows that the occurrance of internal waves observed in reality cannot be explained in a similar way as the wave system generated by a ship in the free surface.

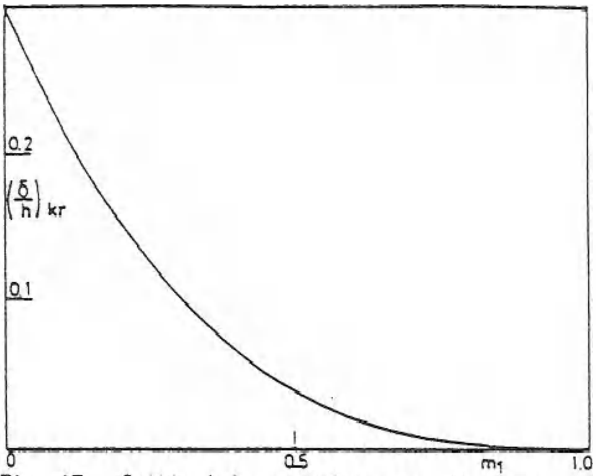


Fig. 13. Critical layer thickness. Maximum layer thickness for the appearance of an interface jump.

## DISCUSSION

Due to the assumptions and simplifications made in the theoretical developments described above, the calculation method certainly has many shortcomings. Several among these are caused by the one-dimensional character of the theory :

- Problems in waters of considerable width cannot be handled, as the blockage factors tend to zero. A semi-empirical "equivalent width" has to be defined in those cases.
- Especially at low speed, the lateral and vertical components of the flow in the vicinity of the ship's bow are too important to allow a fair approximation by means of a one-dimensional theory. However, the effect of the difference between theoretical calculations and experimental observations on vertical displacements can be neglected.
- At speeds higher than the second critical value, the internal wave front is not perpendicular to the ship's centerline, which is in contradiction with the assumptions of one-dimensionality.

Even if the flow had a strictly one-dimensional character, the theory would not be able to take the influence of propulsion into account, or to predict the location of the "interface jump" in the second speed range.

In spite of all these shortcomings, the results of theoretical calculations have shown that the one-dimensional theory is able to give a prediction of the behaviour of both the mud layer and the ship navigating above it. This prediction is fair for the ship's mean sinkage and for interface elevations, and gives a tendency for trim angles. It can be expected that results will be improved if the effect of the propeller(s) on the flow is taken into account, and if the blockage of the lower fluid layer is not neglected.

However, it is useless to refine the presented calculation method before the applicability of the theory on a real mud layer is proven. For this reason, a new test program is planned to be carried out in the Hydraulic Research Laboratory :

- The tests with the scale model of the product tanker will be repeated above a layer of artificially composed mud.

- A similar test program will be carried out with a 1:40 scale model of a suction dredger navigating above a solid bottom, a TCE-petrol layer and a layer of artificially composed mud. As the suction dredger has been used for full-scale tests above a mud layer in the harbour of Zeebrugge, a comparison between reality and model tests will be possible, which is of importance for the selection of a mud simulating material. The full-scale tests mentioned were executed by Decloedt nv and Haecon nv.

- As it is expected that in reality only the upper part of a mud layer is affected by the flow due to the ship's speed, it is of importance to know the position of the lower boundary of this "active zone". In order to acquire more information about the characteristics of the lower boundary mentioned, tests are planned in the Laboratory of Hydraulic Research with a natural mud layer with thickness 0.30 to 0.60 m. It is the purpose to detect the active zone of the mud layer, and to check whether the theoretical conditions for the appearance of a hydraulic jump in an interface between two fluid layers are also valid for the interface between water and mud.

Finally, it can be stated that the study of the interface and of the flow velocities of both water and mud can give an explanation for many phenomena concerning the behaviour of a ship navigating above a mud layer. One of these phenomena, the vertical motions of the vessel, has been handled in this paper, but it can be expected that the influence on the rpm-speed curve is related to the presence of an interface jump (second speed range) or a interfacial stern wave (third speed range) as well, for an interface elevation implies a higher relative velocity between the ship's hull and the water, which not only causes an increase of viscous resistance, but also affects the prestations of the propeller(s). Moreover, an interfacial wavemaking resistance term will have to be added to the total ship's resistance.

It is clear that this paper does not intend to give a final solution for all problems concerning the performance of ships in muddy areas, even not for those considering only vertical motions. It is only the purpose to contribute to a better insight into the nature of the physical mechanisms which are responsible for the phenomena considered.



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