Data-driven reduced order modelling of tide-induced sand bars in confined channels

Tjebbe Hepkema[1], Huib de Swart[1], and Henk Schuttelaars[2]
[1]Institute for Marine and Atmospheric research Utrecht (IMAU), Utrecht University, Utrecht, The Netherlands
[2]Delft Institute of Applied Mathematics (DIAM), Delft University of Technology, Delft, The Netherlands

Tidal bars are bed forms in tidal channels that have a wave-like structure in both the along-channel and cross-channel direction. They are found in tidal channels all around the globe, for example, in the Western Scheldt in the Netherlands, the Exe Estuary in England, the Ord River Estuary in Australia and the Venice Lagoon in Italy. Typically, tidal bars are several meters high, have wavelengths of 1-15 km and migration speeds of meters per day. Understanding their dynamics is important as they are invaluable for many living organisms (e.g., migrating birds) but they hamper marine traffic.

It has been shown, by means of a linear stability analysis, that these bars emerge due to inherent feedbacks between the tidal currents and the erodible bed. When the bars mature, their dynamics becomes nonlinear. Schramkowski et al. (2004) applied a bifurcation analysis to analyse the bar dynamics, but their method was limited to small bottom friction. Here, we developed a numerical (time integration) model that simulates the nonlinear dynamics and the corresponding (stable) equilibrium patterns for realistic parameter values.

Using the output of the numerical model we derive a reduced order model with a method called SINDy (Brunton et al., 2016). Loiseau and Brunton (2018) showed that from output of complex numerical models simulating fully nonlinear fluid flows, SINDy can identify small systems of equations which govern the complex flows. Here we show that, for parameters regimes where the dynamics is weakly nonlinear, SINDy finds a Landau type equation that reproduces the tidal bar dynamics well. The Landau equation is a nonlinear ordinary differential equation in terms of the Fourier amplitude of the pattern that initially has the largest growth rate. The form of this equation corresponds with the one that is expected from the symmetry of the patterns. Also, the application of SINDy to the fully nonlinear dynamics of tidal bars will be discussed.

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