SKIN FRICTION RESISTANCE DERIVED FROM WAKE MEASUREMENT.

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Summary.

This paper gives the results of experiments to measure the frictional resistance of two smooth plank surfaces. The plank surfaces were 50 feet and 18 feet long respectively, precautions were taken to ensure turbulent boundary flow and the resistance was determined by the momentum drag method as described in R and M 1688 of the Aeronautical Research Committee, London, 1935-1936.

The results show that the extent of the wake did not vary with speed and that the depth of the wake below the keel was substantially equal to the width of the wake. This width did not vary with draft.

The momentum distribution curves indicate that with increasing immersion the limiting specific frictional resistance tends to a maximum.

The integrated values of specific frictional resistance do not show a variation in terms of draft within the limits of accurary of the experiments.

These results are compared with the turbulent friction lines proposed by Prandtl-Schichting, Schoenherr and Falkner and also with the resistance of the 50′ plank tested by W. Froude.

At the higher Reynolds numbers the curve from the results in the paper is 5 % higher than te Schoenherr line, 2 % higher than the Prandtl-Schichting line and 1 % lower than the Falkner line.

Introduction.

In the calculation of ship resistance from model data, surface friction resistance has been assessed for many years by the Froude method, i.e. using the results of W. Froude's

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plank friction data as interpreted in the O_m and O_s values by R. E. Froude.

In recent years there has been a move in some quarters to base the surface friction calculation for model and ship on the Prandtl-Schlichting or the Schoenherr lines for turbulent flow, with allowance for the roughness of the ship surface. The extent of laminar flow on an important range of merchant ship models has also been explored and the importance of using a suitable means of stimulating turbulent flow on the model has become evident.

These matters were discussed at the International Conference of Ship Tank Superintendents in London in 1948, and certain interim conclusions were arrived at. Since that date, several aspects of the problem have been investigated at the Ship Division of the National Physical Laboratory, London, and the present paper deals with the results of some of these tests.

The particular tests dealt with here concern the frictional resistance of two plank surfaces 50 feet and 18 feet long respectively.

Steps were taken to ensure turbulent flow on these planks, and the frictional resistance has been determined by the momentum drag method. The method as applied in aerodynamic research is described in R and M 1688 of the A.R.C. 1935-1936, and the same method has been used in this work. In addition to the resistance data obtained, the method gives valuable data on the momentum and velocity distribution over the wake of the plank. It has the further advange that a comparatively thin plank can be rigidly attached to the towing carriage and stiffened against yaw. The disadvantage is the enormous amount of time and analysis necessary to achieve the final answer.

Description of 50' Plank and Apparatus.

As stated above, two planks were tested, most of the experiments being carried out on the plank 50 feet long. The dimensions of this were : length 50 ft., depth 62¼ inches, and thickness 1 inch. The central portion was parallel sided for a length of 45 ft. and each end tapered over a length of 30 inches from a thickness of 1 inch to a thickness of 1/16 inch, the actual ends being circular of 1/32 inch radius.

The section of the plank was a rectangle of 1 inch thickness over the whole depth with the exception of the bottom 3 inches which tapered from 1 inch to a circle at the keel of ¼ inch radius.

For the main body of the plank, multiply wood 1 inch thick of Admiralty marine specification was used. The keel, which was 4½ inches deep, consisted of two lengths of extruded aluminium alloy scarfed together at amidships. The ends were machined from aluminium alloy plate 1 inch thick, and a great deal of trouble was experienced in keeping the thin end of the taper straight. Transverse stiffness was obtained by a girder along the top built up of an 8 inch wide steel plate ¼ inch thick and two duralumin angles 2½ inches × 2½ inches × ⅜ inch thick. After assembly bad places were stopped and filled and the whole surface covered with 3 coats of a phenol resin which was sprayed on, a smooth finish being obtained by rubbing down with wet and dry sandpaper. Owing to the difficulty of handling such a large unbuoyant plank it was originally intended to leave it in the water until the whole investigation was finished, but after 3 days immersion blisters began to appear all over the surface, the joints between metal and wood being particularly bad. After reconditioning, experiments were resumed, the plank being taken out of the water every other night. Measurements of the surface were taken by the Metrology Division of the National Physical Laboratory, and are given in Appendix I.

To ensure fully turbulent flow, a row of pins 1/8 inch diameter and 1/10 inch long were stuck to each side of the plank on a vertical line ½ inch from the bow, the spacing of the pins being 1 inch. A few experiments were also carried out with trip wires .036″ diameter replacing the rows of pins. Both these methods of stimulating turbulent flow have been proved at the N.P.L.

As the carriage which was to carry the plank was only 20 ft. long, it was necessary to build two auxiliary carriages each 15 ft. long to be fastened to the forward and after ends of the main carriage to enable the plank to be rigidly supported over its whole length. Traverse movement of the keel was prevented by fitting straining wires at amidships and also at a distance of 15 inches from the bow. The wires were seven stranded .05 inch diameter stainless steel. Before running, the centre-line of the plank was set up by gauge, parallel to the running rails of the tank and tests were made at several points along the length for alignment.

To explore the velocities and pressures in the wake, 100 pitot tubes (outside diameter .08 inch and inside diameter .06 inch) arranged in 4 rows of 25 each, the distance between the rows being 16,15 and 15 inches, were housed in a rigid framework. The pitot tubes in each row were spaced with one on the centre-line and then at the following intervals on both sides of the centre-line : 0.6, 1.2, 1.8, 2.8, 4.1, 5.4, 6.7, 7.5, 8.2, 9.0, 9.8 and 10.4 inches. Part of the framework was attached rigidly to the plank and served as a guide to allow vertical movement to the part housing the pitot tubes the whole being positioned so that the mouths of the pitot tubes were in a transverse plane 6 inches aft of the after edge of the plank with the middle tube of each row behind the centre-line of the plank. The pitot tubes were connected to copper tubes 3/16 inch internal diameter which were led up inside the port or starboard leg of the framework, these copper tubes being in turn connected by rubber tubing to a 100 glass tube manometer. An eccentriccally mounted steel roller fitted just below the manometer board enabled the experimenter to « freeze » the water levels in the tubes whenever required. This was very necessary as 6 or 7 runs were needed at some speeds before the heads reached their correct levels, due to the fact that the big resistance of the plank (about 430 lbs. at 20 feet per second) and the extra weight of the auxiliary carriages reduced the acceleration of the run.

The general arrangement of the plank and pitot tubes is shown in fig. 1.

Experimental procedure.

Total head measurements were taken using as datum a static head tube clear of the wake in each row of 25 tubes. At the end of each run, the levels were held in position by using the eccentric roller, and the readings written down on previously prepared tables. This method introduces very small errors due to the fact that the internal bore of the glass tubes was not exactly the same in all cases. Previous tests showed that such errors were negligible. The alternative method of photographing the levels was rejected because of the time necessary for enlarging and analysing, time being a very important factor when dealing with such an enormous amount of work as this investigation entailed.

Experiments were carried out at three drafts, viz. 57,41 and 25 inches, the immersions of the pitot tubes being varied in five stages from two inches below the water line to $8\frac{1}{2}$ inches below the keel.

The speed range was from about $2\frac{1}{2}$ feet per second to about 20 feet per second, covered in 11 steps, 5 experiments being necessary to explore the wake at any one speed.

For speeds of from $2\frac{1}{2}$ feet per second to 5 feet per second, the manometer board was inclined to the vertical to give artificially increased heads, but was vertical for the higher speeds.

The whole process was then repeated with static heads pushed over the total head tubes.

The chief causes of error in the readings were :

1) The levels in the tubes fluctuated due to the turbulence in the wake. This fluctuation was not noticeable in the outer tubes nor in the tubes very near the centre of the wake. It was roughly $\frac{1}{2}\%$ of the total head clear of the wake, amounting to .25 inch at 14.78 feet per second.

2) Through the necessity of running « booster » runs to get steady tube readings a surge was set up in the tank water, and although intervals of 40 minutes were made before the final recording run the slow surge was still sometimes giving as much as .2 inch difference in height between its top and bottom levels.

Theory of the Method.

The method used to obtain the resistance coefficient from these measurements is that given in Reports and Memoranda of the Aeronautical Research Committee N° 1688 1935-1936, viz :

$$D = \rho \int\int u_1 \cos \Theta \, (U - u) \, d\,a, \qquad (1)$$

where D = total drag, and ρ = density of fresh water
U = velocity in the undisturbed stream
u_1 = velocity at the point of measurement
u = velocity at zero pressure in the same streamline

Θ = angle of incidence of flow to the plane of the traverse.

This is so small that cos Θ may be taken as = 1.

da_1 = small cross sectional area of wake

All velocities are measured relative to the plank.

The following quantities are obtained from the experiments:

G_0 = total head in undisturbed stream

G = total head at point of measurement

P = static head at point of measurement (assumed to be zero in undisturbed stream)

so that $G_0 = \frac{1}{2}\rho U^2$

$G = \frac{1}{2}\rho u_1^2 + P$

and $G - P = \frac{1}{2}\rho u_1^2$

If we let $\frac{G}{G_0} = g$ and $\frac{P}{G_0} = p$, subsituting in (1)

gives $C_{f1} = 2\Sigma \sqrt{g - p}\,(1 - \sqrt{g})\,da_1$ = local C_f

The drag determined by this method is the profile drag, i.e. it includes friction and form drag, but does not include drag arising from the formation of gravity waves at the free surface.

To obtain the friction drag, the form drag must be subtracted from the total calculated. The form drag is largely dependent on the ratio of thickness to length of plank and an estimate of its value may be obtained from R and M 1838 af A.R.C. 1938.

In addition a correction must be made for the effect on skin friction of the change in local velocity due to the beam of the plank.

A further small reduction in drag must be made to eliminate the resistance of the trip wire or the pins as the case may be.

The actual values of these corrections are dealt with later in the paper.

Young and Maas in Reports and Memoranda N° 1770 of the Aeronautical Research Committee 1937 show that the centre of pressure across the mouth of a pitot tube placed in a stream with a transverse velocity gradient is not at the geometrical centre of the tube. A correction embodying this fact was applied to the integrated results.

DISCUSSION OF RESULTS.

Transverse Distribution of $2\sqrt{g-p}\,(1-\sqrt{g})$.

Fig. 2 shows the change of $2\sqrt{g-p}\,(1-\sqrt{g})$ as the distance of the measuring point from the centre-line of the plank varies. Thirteen curves are show, each curve representing results at 11.2 feet per second but at different immersions of the pitot tubes. Similar sets of curves were obtained at all speeds and drafts. Although readings were taken on both sides of the plank, results are shown for one side only, the curves and spots representing the mean of the port and starboard sides. The actual differences between the two sides were very small. It will be noticed that the width of the wake belt remains constant until the wake completely disappears, the decay being due to the slowing down of the velocities over the central portion. Each of these curves was integrated to give the Cf_1 per unit depth at the respective immersion.

Vertical Distribution of Cf_1.

The local Cf_1 values were then plotted at their correct $\frac{\text{immersion of pitot tube}}{\text{draft of plank}}$ value together with similar results at five other speeds to give fig. 3. This diagram therefore shows the variation of Cf_1 with depth below the water surface and also its variation with speed of advance. As regards the former, attention is drawn to the limited vertical portion which gives constant Cf_1 values. The bottom edge appears to affect the flow as high up as $\frac{\text{immersion}}{\text{draft}}$ of about .6 and the effect of surface tension extends down to about $\frac{\text{immersion}}{\text{draft}} = .2$. The kink in the curves in the region of $\frac{\text{immersion}}{\text{draft}} =$ .9 to 1.1 is probably due to the rather sudden decrease in the induced velocity in the wake as the stream lines expand under the keel.

The effect of change of draft of the plank on the vertical distribution of Cf_1 values is shown in fig. 4. Within the limits of experimental accuracy the wake extends the same actual distance below the keel at all drafts, this distance being equal to the horizontal thickness of the wake belt (as near as this can be measured with the velocities at the outer edges of the wake changing so slowly).

In fig. 4 the vertical portions which show constant Cf_1 values over part of the plank depth, show different values for the three draft. This indicates that in addition to the falling away in the Cf_1 curve below $.6 \dfrac{\text{immersion}}{\text{draft}}$ ratio already referred to, the edge effect extends over the whole depth of the plank.

The momentum in the wake below the keel must be supplied from the rubbing of the water on the plank above. The experiments show that this momentum is substantially constant at the three drafts tested. Therefore in the case of the smallest draft, the rubbing surface being less than in other cases, there will be a bigger relative drain of momentum from the the main body of the wake, resulting in smaller wake velocities in this region and consequently a smaller local Cf_1.

If the value of Cf_1 in the region where the curves are vertical is plotted to a base of draft it will be seen that Cf_1 is tending rapidly to a limiting value somewhat higher than that for the 57″ draft. The experimental evidence is not sufficient to establish this value which would appear to be that for an infinitely deep plank.

Velocity Distribution.

In fig. 5, values of $\dfrac{\text{velocity at measuring point}}{\text{velocity in undisturbed water}} = u_1/U$

are given for thirteen immersions of the pitot tubes. Contour lines are drawn for constant values of u_1/U, the effect of the eddies at the bottom edge being shown by the distortion of the contours in this region. This diagram shows once again the constant width of the wake until its final disappearance. It also shows that u_1/U at the centre of the plank diminishes slowly from $\frac{\text{immersion}}{\text{draft}}$ values of .07 to .85 but much faster from this point downwards.

It may be assumed that the law of the velocity distribution curve at any immersion is of the form $u_1/U = A\,(y/d)^n$

where : y = distance of measuring point from centre-line
d = width of wake belt
and n and A are constants.

Values for n and A were obtained for three portions of each curve, one near the centre-line, one near the edge of the wake and the other between these two extremes, and finally a mean of these three values was taken as a good approximation to the law of the curve. The following table gives values for the 41 inches draft condition at 11.2 feet per second.

$\frac{\text{immersion}}{\text{draft}}$	.072	.171	.258	.356	.463	.561	.658	.756	.830	.927	1.025	1.123
n	.151	.139	.141	.133	.132	.133	.139	.125	.118	.118	.074	.060
A	1.047	1.018	1.013	1.003	1.005	1.005	1.019	1.018	1 026	1.054	1.016	.988

The above values for n may be compared with the value of 1/7 usually assumed by Prandtl in his formula $\frac{\mu}{U}=\left(\frac{y}{d}\right)^{\frac{1}{7}}$

Ref. : Z. Angero, Math. & Mech. 1925.

Experiments with an 18 feet Plank.

Similar experiments were also carried out on an 18 feet long plank, the dimensions being — length 18 feet, depth 29 inches, thickness for the top 11 inches was 3 inches and from this point it tapered to the keel which was a circular arc of ¼ inch diameter. The ends were tapered for a length of 3 6inches finishing in a circular arc of 1/32 inch radius. It was constructed of aluminium sheets 1/10 inch thick screwed to duralumin frames 1 inch thick 2 feet apart. The keel and ends were made of aluminium alloy castings. The whole surface was painted with several coats of a cellulose enamel.

The momentum drag experiments were carried out at two drafts, 23 inches and 15 inches, the integrated results of which are shown in fig. 6.

Considerable wave-making was caused by this plank and as a result a correction had to be applied to the readings of head. It was noticed that the readings clear of the wake were different from those obtained in open water at the same speed. The assumption was made that this difference was caused by the orbital motion in the wave and that this effect was unaltered over the width of the wake. The difference in head of the outside tubes compared with their head in open water at the same speed enabled the orbital motion speed to be found and the head of each tube was then corrected for this amount.

The resistance of this plank was measured directly on a dynamometer and the wave resistance was calculated making an arbitrary correction for viscosity. The frictional resistance of the plank arrived at in this way was some one to two per cent higher than that deduced from the wake momentum method and this was considered to be a reasonable check on the results.

Cf for the whole plank.

Integration of the curves such as those shown in fig. 3 was carried out at all the speeds run, and when divided by the area of both sides of the plank gave the friction coefficient Cf for the whole plank

i.e. $$Cf = \frac{Rf}{\frac{1}{2}\rho A U^2}$$

where Rf is the skin friction resistance in lbs.

ρ is the density of fresh water

A is the area of 2 sides of the plank in square feet

U is the speed of advance in feet per second.

The resistance of that part of the four straining wires which lay in the region over which the momentum drag measurements were carried out was assessed from published data and subtracted from the Cf values as calculated.

Fig. 6 shows Cf values plotted on a base of Reynolds number, $\frac{UL}{\nu}$ where:

L is the length of the plank in feet

and ν is the kinematical viscosity of fresh water.

The results are plotted for the 50′ plank and also for the 18′ plank. These results have been corrected for the resistance of the turbulence stimulators in each case, the correction being based on data obtained in special tests at the N.P.L.

In the case of the 50′ plank this correction amounts to .00002 for the pins and .00004 for the trip wire. This difference between the resistance correction for pins and trip wire is roughly in line with the difference between the corresponding spots in fig. 6. The corrected curve was obtained by subtracting the .00002 value.

In the case of the 18′ plank the correction for trip wire amounts to .00005 and this has been used in fig. 6.

To arrive at the frictional resistance for a flat plank surface a correction should be made for form effect (including velocity effect). For fine forms this correction expressed as a fraction of the profile drag is probably of the order of two to three times the ratio $\frac{\text{(mean maximum thickness)}}{\text{length}}$. This gives about one third of one per cent for the 50′ plank which may be neglected, but gives two to three per cent for the 18′ plank. This correction has not been applied to the 18′ plank.

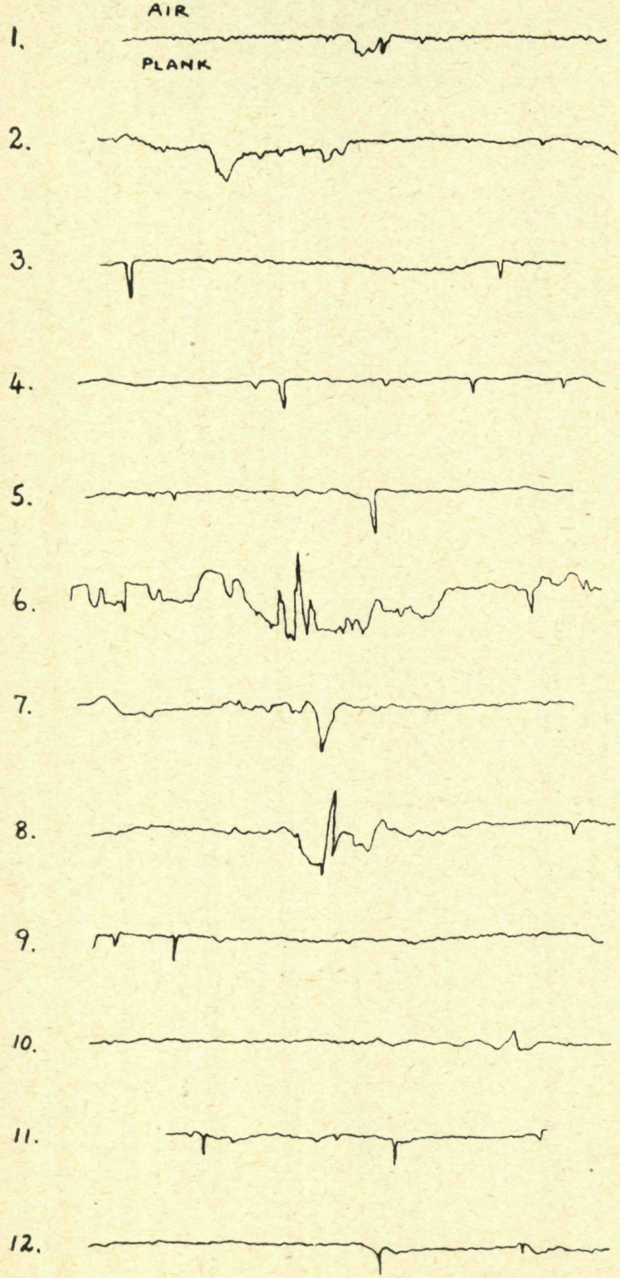
It is considered that the variation of the spots in terms of draft for either plank are generally within the limits of accuracy of the experiment results. This conclusion is somewhat at variance with the argument developed above from the local Cf_1 values.

In fig. 7 the corrected curves are shown along with the Prandtl-Schlichting line, the Schoenherr line and the Falkner line. The original Wm. Froude 50′ plank result has also been indicated as a matter of interest although the water temperature of these tests is not known.

It is not intended to enter into a discussion concerning the determination of a minimum turbulent friction line or of the corrections for curvature which must be made to that line in applying it to a ship form, but fig. 7 is of some interest in this connection.

The Falkner line was determined from a wake momentum integration in two dimensional flow and its high position in relation to the data of this paper is in line with the argument

DEFECTS IN SURFACE



based on fig. 4, that for an infinitely deep plank the Cf value would be somwhat higher than obtained in the present tests.

The Schoenherr line which is the one proposed for ship-model correlation at the 1948 Conference of Ship Tank Superintendents appears to be low over the higher range of R_n covered by these tests.

Acknowledgement.

This paper is based on work carried out in the Ship Division of the National Physical Laboratory, and is published by permission of the Director of the Laboratory.

APPENDIX I

The following gives a summary of a report on the surface finish of the 50 feet plank. The work was carried out by

The surface finish of selected portions of the plank was determined by means of plastic replicas subsequently measured on a Talysurf instrument.

The attached chart records which are corrected for replica inversion show on an enlarged scale the movements of the exploring probe of the Talysurf instrument in following the irregularities present in the surface of the replicas.

The vertical magnification used was 4000 so that one small division across the chart grid represents 25 micro-inches (0.000025 inch). The horizontal magnification employed was 50, hence a probe traverse of 0.24 inch is shown by a chart length of 12 inches.

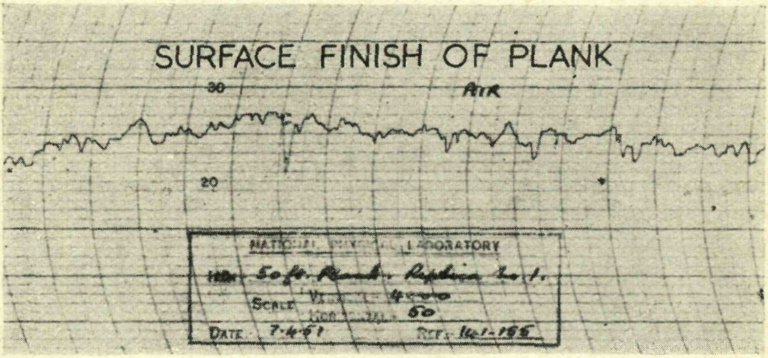
A. The records have been reduced to one half this enlargement in reproducing them for this paper.

Typical defects in the surface were selected and were recorded by means of the Portable Aerofoil Recorder on small smoked glass plates. This instrument provides records at a vertical magnification of 18 and a horizontal magnification of 1/10. The attached photographs, which have been enlarged 5 times, are thus at magnifications of 90 vertical and ½ horizontal.

B. The reproductions in this paper are 0,9 times the size of the original records.

This ends the summary of the report.

It must be remembered that these photographs are not typical of the surface but are enlargements of areas specially selected for obvious defects, occurring chiefly at the joints of wood and metal.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30
AIR
20
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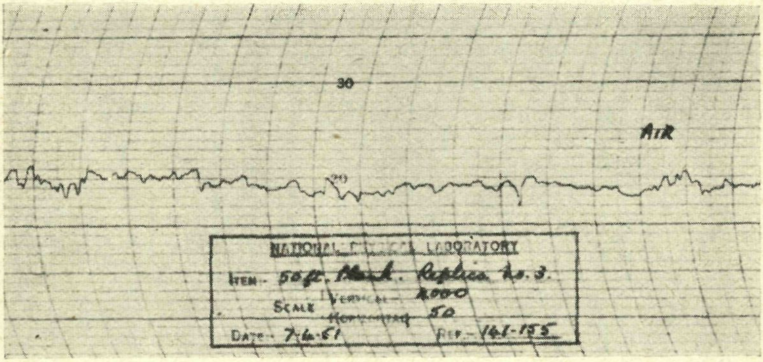
SCALE VERTICAL 4000
HORIZONTAL 50
DATE 7-4-51
REF. 141-155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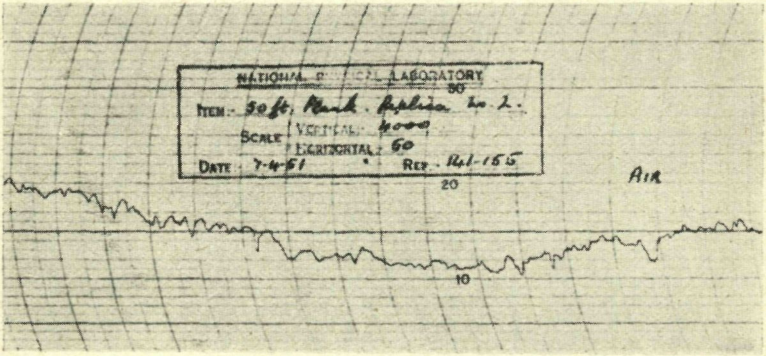
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10

APPENDIX II.

Streamflow Experiments on the 18 feet Plank.

Three separate methods were used in an attempt to determine the extent of laminar flow on the plank. The first two depended on the fact that the boundary layer suddenly thickens at the point of transition causing a big change of velocity. In the first method a hot wire anemometer was used to record this change, but the vibration of the carriage prevented accurate readings of the electrical recording instruments and the method was a failure.

In the second case two pitot tubes were used, one very close to the surface of the plank and the other about an inch away, the pitot tubes being led one to each leg of an inverted glass U tube. Readings of the water levels in the U tube were taken at various speeds with the pitot tubes in a given transverse plane, any big change change in levels indicating the presence of turbulent flow. No consistent results were obtained in this way and it was decided to observe the water levels while the pitot tubes were being slowly drawn aft with the inner tube touching the surface of the plank. No sudden change in levels occurred so that it was decided to abandon this method.

The third method was that used so successfully on models, viz. ink traces from holes in the surface. Holes 1/32 inch diameter were drilled in hull at a distance of 5¾ inches from the bow and at distances of 13, 18 and 20½ inches from the keel. The results from this method were consistent when the experiments were repeated and are given in tables I and II.

An extra hole was drilled 1/32 inch diameter 1¾ inches aft of the shoulder (i.e. 3 feet 1¾ inches aft of the bow) and 13½ inches from the keel, and gave the following results:

Extend of laminar flow aft of hole	Speed in feet per second
12 inches	3
8 inches	3½
5 inches	4
2 inches	4½
nil	5

TABLE I

Height of hole from keel	Speed in ft./sec.	Extent of laminar flow	Remarks
13 inches	3	3 ft. from bow (Shoulder of model)	Stream seemed to break away at shoulder and run as a different stream line 1½ inches nearer the waterline.
	3¼	3 ft. from bow	
	3½	3 ft. from bow	Occasional break down to 9″ from hole.
	4	3 ft. from bow	Sometimes laminar aft of shoulder
	4½	4½ inches from hole	Occasionally laminar to 9″ aft of hole.
	5	4 inches from hole	Always turbulent aft of 4″ from hole.
	5½	1 inch from hole	
	6½	4 inches from hole	
	7	nil	
18 inches	3	3 ft. from bow	Sometimes laminar aft of shoulder
	3½	3 ft. from bow	
	4	3 ft. from bow	
	4½	3 ft. from bow	
	5	3 ft. from bow	Occasional break down to 12″ aft of hole.
	5½	6 inches from hole	
	6	4 inches from hole	
	6½	3 inches from hole	

TABLE II

Height of hole from keel	Speed in ft. sec.	Extent of laminar flow	Remarks
20 ½ inches	3	3 ft. from bow	Flow parallel to keel for 6″, then a sharp rise up to surface for about 6″. then it dips down to original level.
	3½	2 ft. 6 ins. from bow	
	4	2 ft. 3 ins. from bow	
	4½	2 ft. 3 ins. from hole	
	5	6 inches aft of hole	
	5½	5 inches aft of hole	
	6	4 inches aft of hole	
	6½	nil	

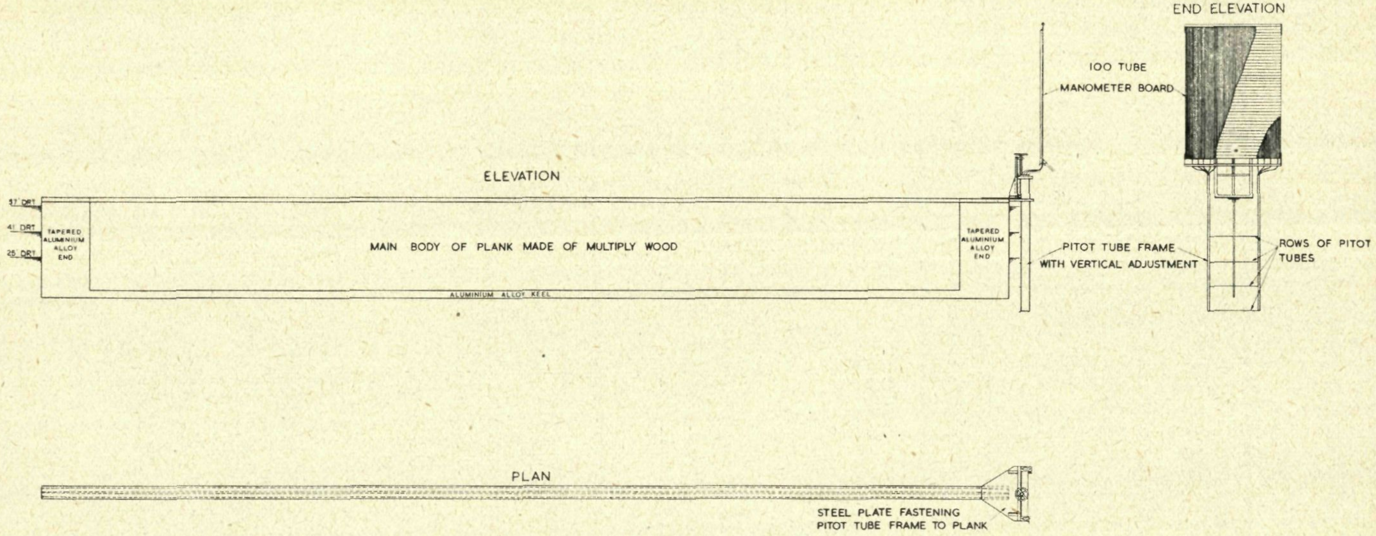

FIG I. GENERAL ARRANGEMENT OF 50 FT PLANK & PITOT TUBES.

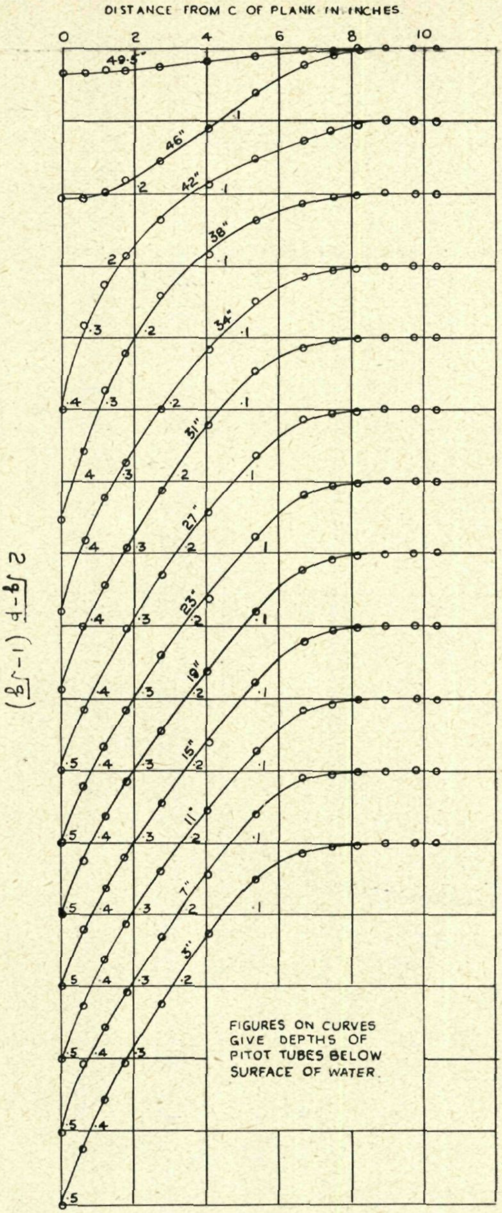

FIG. 2 VARIATION OF $2\sqrt{q-p}\,(1-\sqrt{q})$ WITH DEPTH.

DRAFT OF PLANK = 41"
SPEED = 11·22 FT/SEC

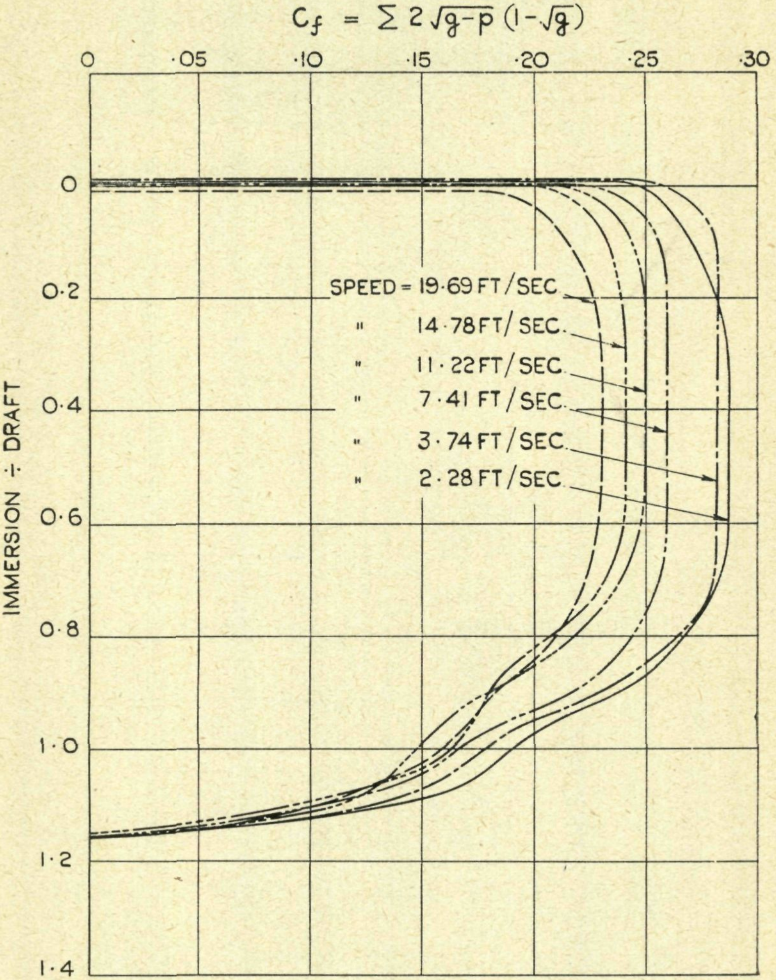

FIG. 3. VARIATION OF C_f WITH SPEED

DRAFT = 57"

(C_f read C_{f1})

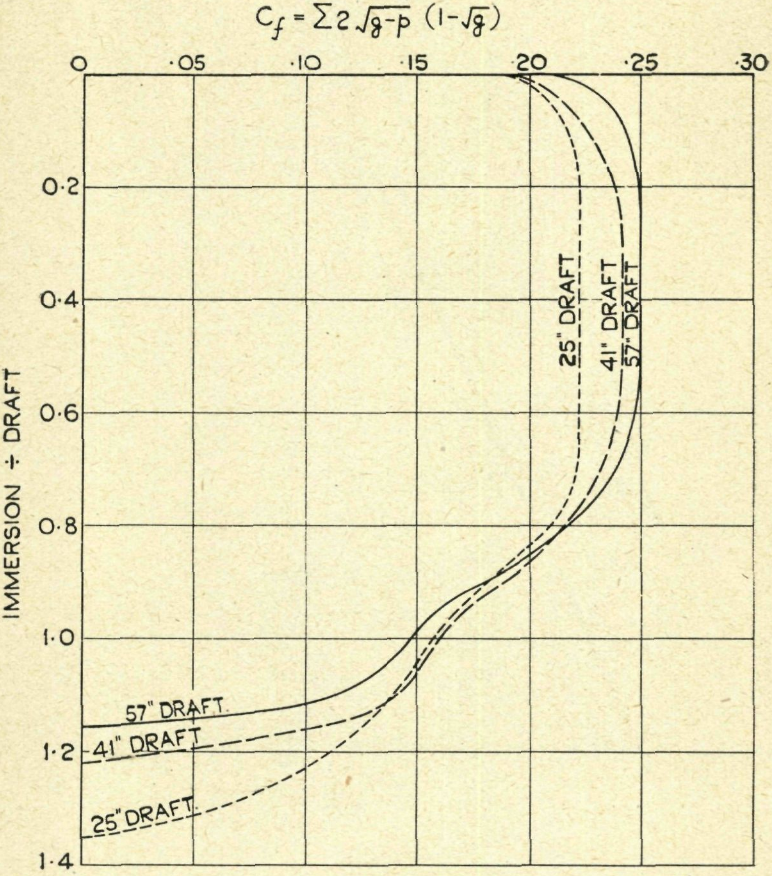

FIG. 4. VARIATION OF C_f WITH DRAFT OF PLANK

SPEED = 11·2 FT/SEC.

(C_f read $C_{f\,1}$)

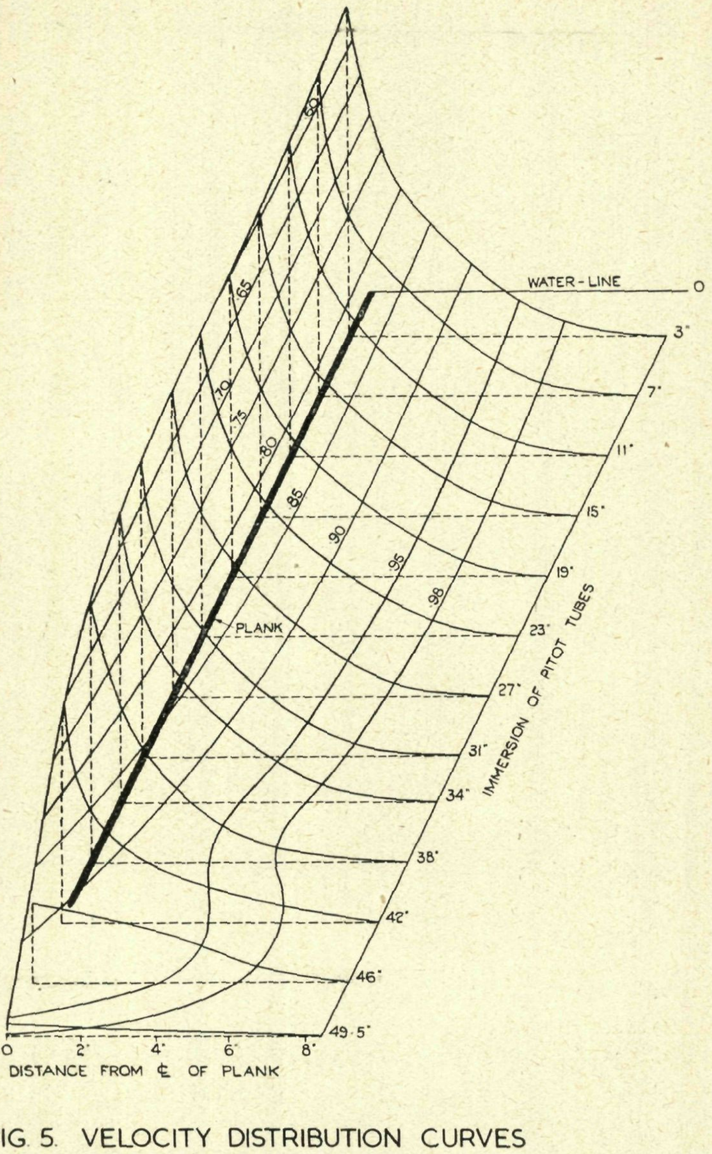

FIG. 5. VELOCITY DISTRIBUTION CURVES AT VARIOUS IMMERSIONS.

DRAFT OF PLANK = 41"

SPEED = 11·2 FT/SEC

FIGURES ON CONTOUR LINES ARE $\frac{\text{VELOCITY AT MEASURING POINT}}{\text{VELOCITY IN UNDISTURBED WATER}}$

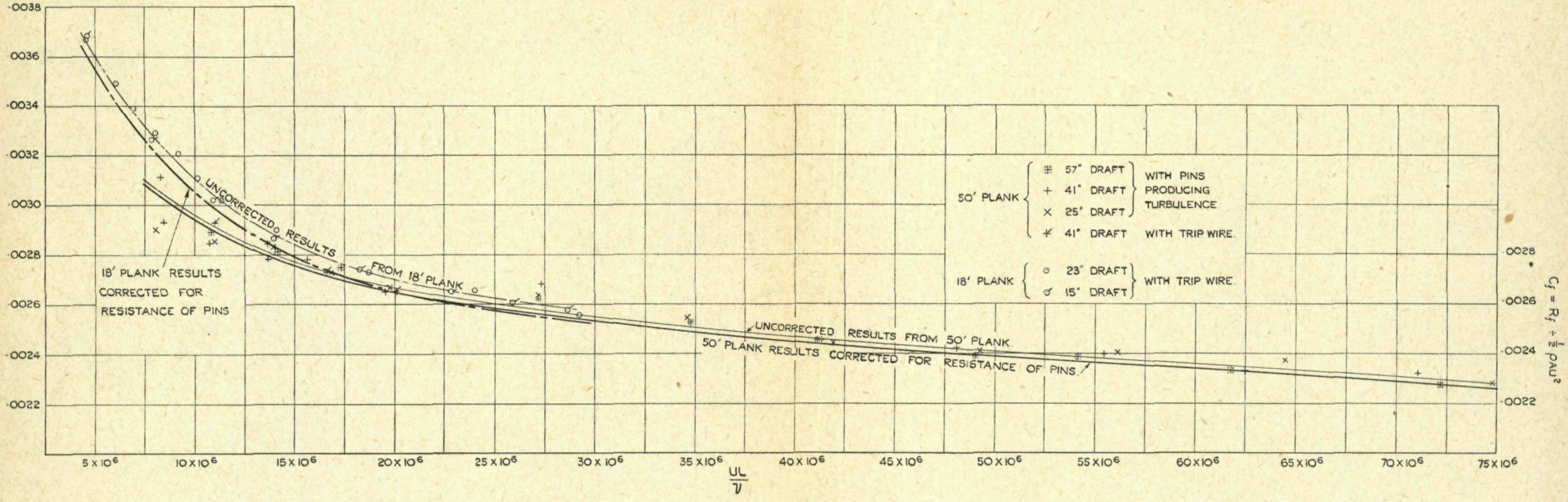

FIG. 6. RESULTS WITH 18' AND 50' PLANKS.

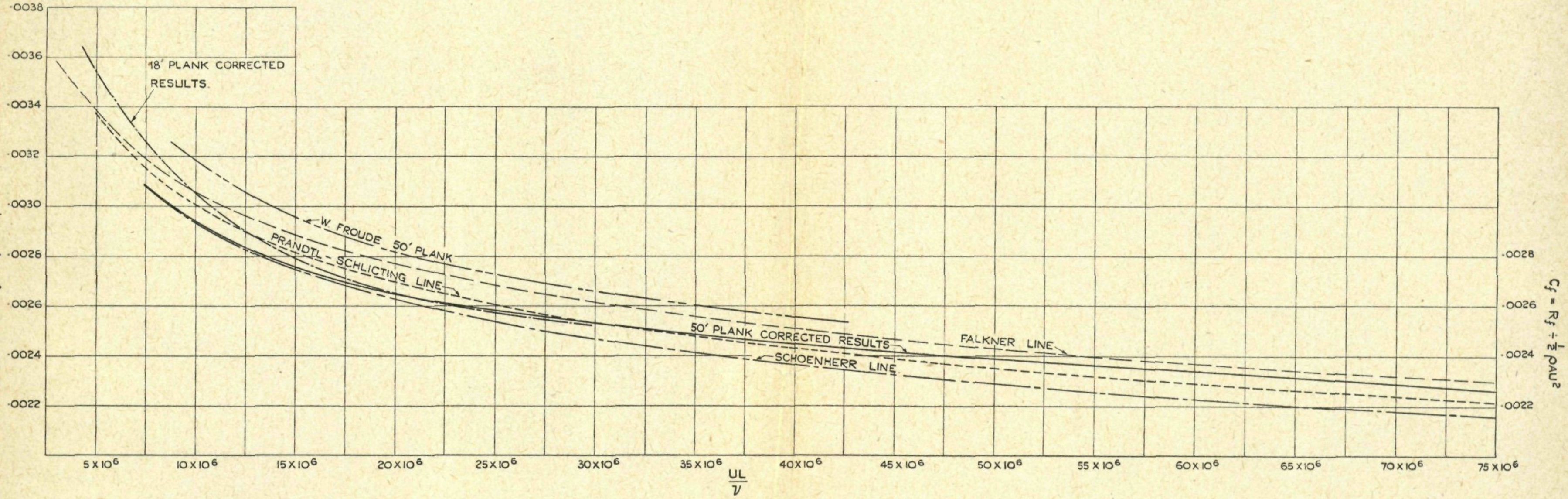

FIG 7. COMPARISON OF 18' AND 50' PLANK RESULTS WITH OTHER DATA

REMARKS BY PROF. G. VEDELER.

I know very little about skin friction. But I should like to ask one or two questions in connection with this important paper.

First of all the conception of the Reynolds Number gives me certain difficulties when applied to turbulent flow. Actually the Number is defined for laminar flow only. When used for turbulent flow, it seems to be a kind of an artifice. My first question is this : Will the fitting of a trip wire or another turbulence producing device not be equivalent to a kind of an artificial lengthening of the plank and thereby to an equivalent alteration of the Reynolds Number ?

My other question concerns Fig. 4 of the paper. At the section in question (just behind the plank) a wake width of about 8 1/2 inches has been measured both horizontally and vertically below the plank. If one goes to the limit of zero thickness of plank, will then the wake below the keel disappear ? In case, should this not mean that for zero thickness of plank the parts of the curves in Fig. 4 which lie below the immersion/draft ratio = 1,0 wuold disappear, and the specific resistance should be calculated only from the parts of curves above the 1,0 line.

Contribution de M. L. F. G. SIMMONS.

The experimental results displayed graphically in Figs. 3 and 4 reveal an unexpected distribution of momentum loss, measured in the wake at different depths. Inspection shows the curves to possess, qualitatively, certain features in common; namely, an initial increase in the magnitude of Cf, with depth until an immersion/depth ratio of about 0,2 is reached; followed firstly by a uniform part, roughly between ratios 0,2 to 0,6 where Cf, remains uniform, and afterwards by a part below 0,6 where Cf, decreases to zero, in all cases at about 9 in. below the lower edge of the plate. Considering the reasons for this kind of distribution, the authors imply that the effects of surface tension between air and water extend down to an immersion/draft ratio of 0,2 and suggest that the shape of the curves in the region 0,9 to 1,1 is probably due to changes of velocity induced in the wake as the stream lines expand under the keel.

It is, of course, conceivable that such an expansion could be brought about by the pressure system over the keel, since the net effect would be an upward force, the reaction of which would impart downward momentum to the water. Calculations based on data derived from tests with the 50 ft plank in fact show that only a small force — equivalent to an average pressure over the keel of $0,002 \rho U^2$, ρ being the density of water and U the speed — would be needed to deflect the streams to the extent observed. But without further evidence, it cannot be established that the pressure distribution could provide even the small force required.

Another and perhaps more effective agency in contributing a downward movement to the surface flow arises, it is suggested, from diffusion caused by turbulent motion. Familiar examples of the effects of this mechanism are to be seen in the explanding wake of a ship, and in the billowing smoke trail from the funnel. It has long been known to be the means by which momentum is transferred from the high velocity, surface streams carried along by a moving plate, to the outermost limit of the turbulent boundary layer. Its effectiveness in transporting energy outwards is in marked contrast to the slower rate of conduction by viscosity in laminar flow; moreover, turbulence sustains high rates of shear in a very thin, laminar layer of water in immediate contact with the surface, and thus leads to large values of skin-frictional drag. In the ideal case of two-dimensional flow the transfer of momentum would be in a direction normal to the plate; there would be no lateral diffusion between neighbouring elements. Near the lower edge of a finite plate, however, where the high velocity streams in the boundary layer would be in contact with stationary water below the plate, owing to a large gradient of longitudinal velocity along any vertical line, diffusion of turbulence from the boundary layer will take place in a downward direction.

If this process is responsable for the spread observed in the wake, it might be expected also to influence the flow over a thin plate, with a longitudinal edge exposed to the stream. This is found indeed to be the case; for the curves of momentum loss (1), measured in a wind tunnel with a plate 16 ft in length, 6 in wide, and 0,05 in thick, extend into the stream beyond the edge of the plate at each section explored back from the leading edge. On examination the curves for positions near the leading edge for some 3 ft back, are seen to bear a close resemblance to those of Figs. 3 and 4; the lateral spread, expressed as fraction of the half width of the plate, however, is larger than that found in the water experiments. Since turbulence in the general stream is also a factor in the problem, as is clearly shown by the variable dispersion of smoke at different lapse rates in the atmosphere, there can be little doubt that the results obtained in the wind tunnel were influenced by the high degree of micro-turbulence known to have been present in the air current.

Various mathematical theories have been applied to problems of turbulent diffusion; and one, Taylor's correlation theory (2), has been successfully adapted (3) to predict the vertical diffusion of smoke emitted from a continuous line source placed on the ground and extending infinitely across the wind. Superficially, this problem is similar to the present one, but further developments of the theory are needed to deal with the more difficult case of continuous generation of turbulence over a surface.

In the absence of theory, speculations on the consequences resulting from a downward expanding wake may not be out of place. Firstly, the imposed vertical motion probably would not be confined locally to water passing the keel but would extend to regions in the boundary layer both above and below. The stream lines which can be drawn in these regions to represent the average direction of flow at any point, will therefore be inclined downwards to a degree dependent on the distance from the keel. To make conditions clearer, consider the motion relative to a plate of deep draught held stationary in a stream of fluid, and imagine horizontal tubes of flow of equal width and area packed above one another along the leading edge. Neglecting the effects of surface tension and wave motion, the tubes at the top and for some depth below the water surface will remain of constant width during their progress downstream, but will expand outwards from the plate under the retardation due to skin friction. Those at a lower depth, however, will be subject te vertical, al well as lateral, expansion. Now if the results for the 57 in. draught represent approximately the values of the coefficients for two-dimensional flow, then the stream tubes down to an immersion/dept ratio of about 0,6 must remain horizontal and of constant width, whilst those below expand vertically by amounts varying with depth. In order to account for the smaller Cf, values measured at draugts 25 in. and 41 in. it is necessary to suppose that between ratios of 0,2 to 0,6 the stream tubes assume a fan-shaped pattern due to the slight expansion of each individual element.

According to the measurements, between the limits of the two ratios above, the value of Cf, remains unchanged but different for each draught. In interpreting the results it is, of course, important tot remember that, with the type of flow envisaged, Cf, at any given depth is not a measure of the local Cf since the momentum theorem only affirms that the summation of Cf, over the wake provides a measure of the skin-frictional coefficient of the plate. That the aggregate result for each of the three draughts is approximately the same suggest that the shearing stresses in the laminar sub-layer remain unaffected by the vertical diffusion of the turbulent layer.

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(1) Stanton T. E. & Marshall D., Journal of the Institution of Naval Architects, 1924.

(2) Taylor G. I. Proceedings of the London Mathematical Society (2), 20, 1922.

(3) Sutton O. G. Proceedings of the Royal Society, A, 182, 1932.