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Hydrodynamic models for very shallow coastal seas

Application to the Belgian coast

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Introduction

In the past, hydrodynamic models successfully reproduced the sea surface motion induced by tides and storm surges. Unfortunately, the simulation of horizontal currents was not very satisfactory because the discretization of the coast and of the bottom topography was too crude (spatial steps of about 30 km).

To improve the quality of the calculated current pattern, the numerical representation of the geometry must be refined : sand banks, beaches, flood and ebb channels have to be taken into account in the model.

Another application of hydrodynamic models is the prediction of dynamic effects of coastal engineering projects. A refinement of the grid allows a better resolution of the horizontal and vertical geometry.

The simulation of tides in a very shallow region with an irregular topography requires suitable and accurate numerical schemes. In this paper, two hydrodynamic models of the Belgian coast are presented. The first one is based on an explicit scheme, the second one on a semi-implicit scheme. Comparisons with the observations are made, and different boundary conditions at the Western Schelde mouth are tested.

1.- Mathematical formulation of depth-averaged hydrodynamic models

The equations of a depth averaged model are (e.g. Nihoul, 1975) :

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}(H\bar{u}) + \frac{\partial}{\partial y}(H\bar{v}) = 0 \qquad (1)$$

$$\frac{\partial (H\bar{u})}{\partial t} + \frac{\partial}{\partial x}(H\overline{uu}) + \frac{\partial}{\partial y}(H\overline{uv}) = H\left(f\bar{v} - g\frac{\partial \zeta}{\partial x}\right) - k\,\bar{u}\sqrt{\bar{u}^2 + \bar{v}^2} + \text{disp. terms} \qquad (2)$$

$$\frac{\partial (H\bar{v})}{\partial t} + \frac{\partial}{\partial x}(H\overline{uv}) + \frac{\partial}{\partial y}(H\overline{vv}) = -H\left(f\bar{u} + g\frac{\partial \zeta}{\partial y}\right) - k\,\bar{v}\sqrt{\bar{u}^2 + \bar{v}^2} + \text{disp. terms} \qquad (3)$$

where

$$\bar{u} = \frac{1}{H}\int_{-h}^{\zeta} u \, dz ,$$

$$\bar{v} = \frac{1}{H}\int_{-h}^{\zeta} v \, dz ,$$

$$H = h + \zeta ,$$

ζ is the sea-surface elevation above the equilibrium level, f the Coriolis parameter, g the acceleration of gravity and k the bottom friction coefficient.

The dispersive terms in equations (2) and (3) result from turbulent and shear effects. They can be parameterized as follows :

$$H\nu\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2}\right) \quad ; \quad H\nu\left(\frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2}\right)$$

with ν a coefficient of "viscosity" used to formulate the dispersive terms.

Equations 1, 2 and 3 will be solved with appropriate initial and boundary conditions :

1. initial conditions

   $\bar{u}$, $\bar{v}$ and $\zeta = 0$ at $t = 0$

   for all grid points, except along open sea boundaries where the elevations or currents are prescribed ;

2. boundary conditions

Along the coasts : $\frac{\partial \overline{\underset{\sim}{v}}}{\partial n} = 0$

with $\underset{\sim}{n}$ the normal at the coast.

Along open sea boundaries, the elevations or (and the currents must be given (see § 3).

2.- Main features of the Belgian coastal models

The hydrodynamic models cover the Belgian coast, part of the Dutch coast up to the Eastern Schelde, and are limited in the Western Schelde at Vlissingen. The seaward extension is of about 30 kilometers (see fig. 1).

The mean equilibrium depth is very small (10 meters), and the maximum is $h_{max} \sim 25$ m. The bottom topography is rather unequal : narrow and deep flood and ebb channels, sand banks, etc...

The observations (Van Cauwenberghe, 1973) show that the spatial variation of currents is mainly related to the shape of the bottom, and that the characteristic lengths for $\overline{u}$ and $\overline{v}$ are much smaller than those related to ζ variations. Consequently, the advective terms are of some importance, and must be suitably reproduced espacially when the spatial step (Δx) is small.

3.- Influence of the open sea boundary on the hydrodynamic of the southern North Sea

Daubert and Graffe (1967) investigated existence conditions for linear and non-linear long-waves equations. The linear form of equations (1) to (3) requires only one-point boundary conditions, so only ζ at the open sea boundaries. For the non-linear case, two-points boundary conditions are required when the flow is directed into the region of computation, and one-point boundary conditions when it is directed out of this region.

In the case of a real fluid, the problem of boundary data is a problem of sensitivity of boundary elevations to the internal velocity field. According to Abott et al. (1973) : *If the dynamic effects dominate over resistance effects, two-points data will be desirable over all inflow segments,*

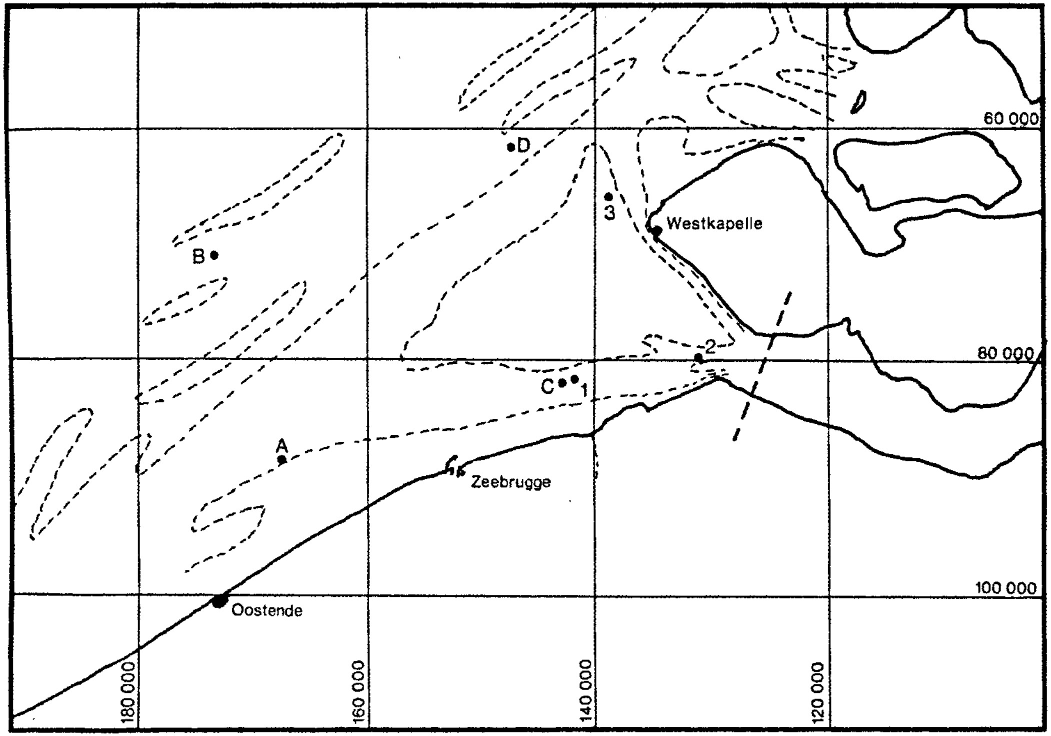

fig. 1.

Positions of the stations of comparison

whereas the dominance of resistance effects implies the practical sufficiency of one-point data throughout.

For the open sea boundaries, the inertial terms are small compared to the pressure and frictions terms : one-point boundary conditions are prescribed and they are derived from other computations and observations. Along these open sea boundaries, equations of motion are assumed to be linear.

In order to visualize the importance of the nature of the boundary condition near Vlissingen on the dynamics of the horizontal and vertical motions, two numerical simulations are performed : the first one with a condition on the sea elevation, the second one with a condition on the water transport. Fig. 2 shows the current roses at some references points (see Fig. 1), the elevation and the flow for a cross section close to Vlissingen (the dotted lines are related to a forced elevation and the solid lines to a forced flow at Vlissingen).

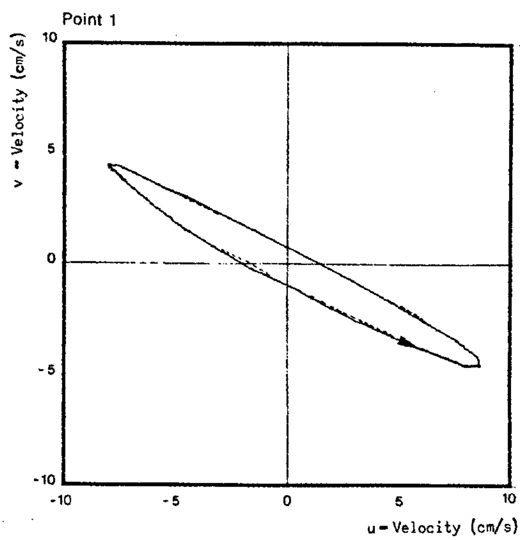

fig. 2a.
Current roses

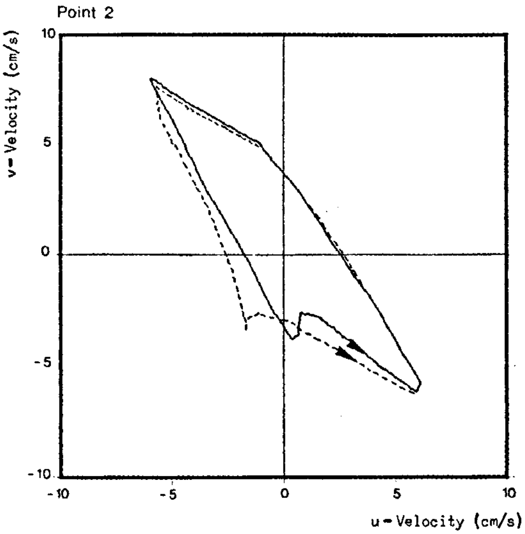

fig. 2b.

Current roses

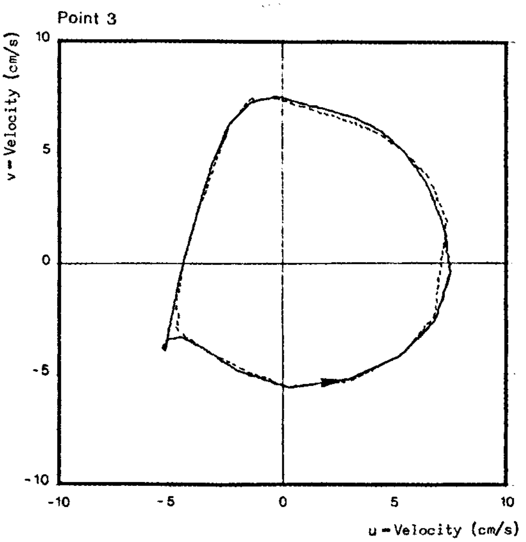

fig. 2c.

Current roses

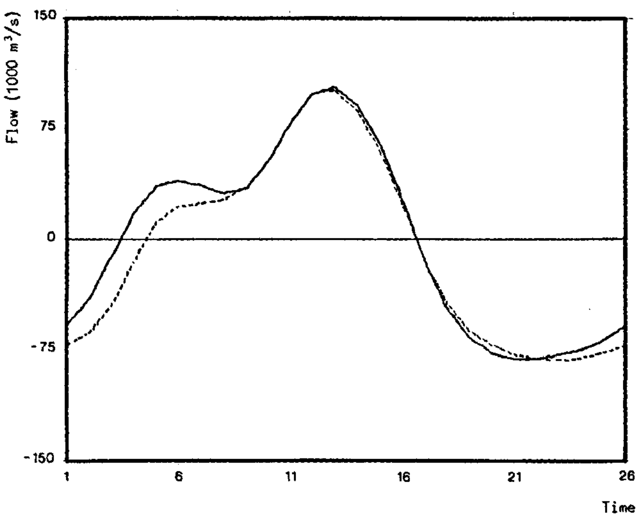

fig. 2d.

Inflows and outflows in cross section close to Vlissingen

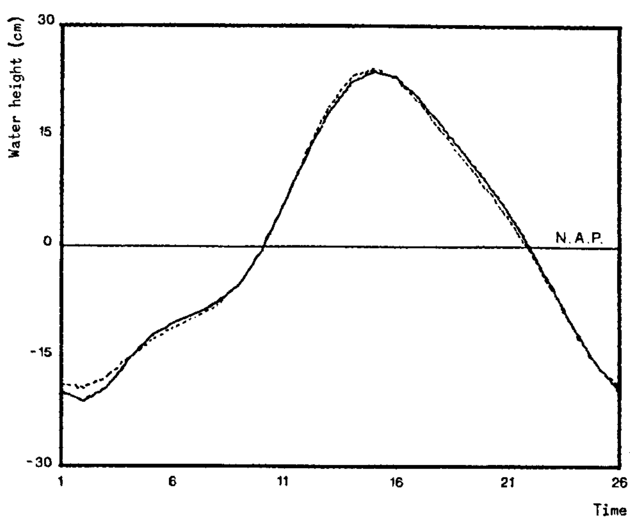

fig. 2e.

Sea elevations at Vlissingen

From these diagrams, one may deduce that the inflows or outflows are more sensitive than the elevations to the nature of the open sea boundary conditions. The Western Schelde influences the velocities in a sector of about 25 kilometers. An analysis of these simulations shows that the best boundary condition in the cross section near Vlissingen must be a double-point condition (a condition on the water transport).

4.- Accuracy of numerical schemes

The previously mentioned schemes had the following truncation errors : for equations with advection terms,

truncation error = $O(\Delta t) + O(\Delta x)$,

for equations without advection terms,

truncation error = $O(\Delta t) + O(\Delta x^2)$.

This accuracy is adequate to simulate long-waves in rather deep-coastal seas (e.g. the North Sea and the Southern Bight), where the advection terms are small compared to the others.

Along the Belgian coast, the topography is very irregular and the depth small. For this reason, the advection and the friction terms generate strong harmonics of the fundamental forced wave.

In a first approach, the discretization of the advective terms is non centered. This formulation induces a numerical viscosity ($\nu_{num} \sim \frac{\bar{u}}{2} \Delta x$) which is larger than that produced by turbulence and shear effect. There results a weak damping and smoothing of the solutions.

As hydrodynamicists are also confronted with practical problems (e.g. modifications of the current pattern due to dams and harbours) one must develop more accurate numerical schemes where the artificial viscosity is weak and the higher harmonics are suitably reproduced.

5.- An explicit predictor-corrector

In 1978, Ronday showed that the first order schemes under-estimate the first harmonic of the semi-diurnal lunar tide. To simulate accurately the higher harmonics of periodic tidal waves and the hydrodynamic perturbations, one must develop a more precise scheme.

As shores and sand banks arising at low tide complicate the resolution of the equations, only an explicit scheme (e.g. Flather and Heaps, 1975) can simulate these phenomena in a simple and "cheap" way. The simpliest scheme that may answer all these requirements is an explicit predictor-corrector.

The predictor is merely the previously used scheme (without the dispersive terms) and its role is the stabilization of the whole scheme. In the corrector step, the discretization of the derivatives are all centered and the viscosity coefficient has a more realistic value. An estimate of the critical time step is

$$\Delta t_c \sim \frac{\Delta x}{\sqrt{2\, g\, h_{max}} + \bar{u}_{max}}$$

This predictor-corrector scheme has been successfully applied to the simulation of tides in the English Channel (Ronday, 1978). Unfortunately, difficulties appeared when it was applied to the Belgian coastal region : a local unexpected instability arose above the Walcheren Island near the open boundary, after one half period of the lunar tide. This instability may be due to over-simplified open sea boundary conditions in this area.

6.- Limitations of explicit and semi-implicit schemes

To improve the quality of the numerical simulations, one must refine the representation of the bottom topography. Unfortunately, some important problems arise from the reduction of the spatial step Δx : memory occupation and computation time.

6.1.- MEMORY OCCUPATION

If x is divided by two, the memory occupation is multiplied by four. As the core of the computer is limited, the refinement of the grid will also be limited.

6.2.- COMPUTATION TIME

For the previous described schemes, the stability condition is very restrictive : the critical time step is proportional to the spatial step. To study the same area, if the spatial step is reduced by a factor two, the computation time is approximatively multiplied by eight. Consequently, the tidal simulation with a precise explicit scheme (predictor-corrector) and a very fine grid becomes very expensive. This smallness of the time step - due to stability requirements - is not fully justified because the characteristic times of $\bar{u}$, $\bar{v}$ and ζ do not need such a temporal accuracy. For this reason, it is attractive to develop semi-implicit schemes.

The main advantage of semi-implicit schemes is the less restrictive stability condition. Leendertsee (1967) developed a semi-implicit scheme accurate to the first order in time and to the second order in space. The instability of his numerical code is due to a particular formulation of the bottom stress. In a first approach, we have developed a simplier semi-implicit scheme accurate to the first order in time and in space, where the stability condition is not related to the bottom stress. The time step must verify the following condition :

$$\frac{\Delta t}{\Delta x} < \frac{2}{|\bar{u}|_{max} + |\bar{v}|_{max}}$$

This computation code is applied to a region covering a large part of the Belgian coast, and the chosen numerical grid is characterized by a small spatial step : $\Delta x = 500$ meters. The open sea boundaries are deduced from the previous predictor-corrector simulation.

To analyse the advantages (or disadvantages) of this new code, the same tide is simulated by means of two codes : the first one is explicit (see § 3), the second one is semi-implicit. The explicit scheme has a time step $\Delta t = 20$ s , the semi-implicit $\Delta t = 200$ s . Figures 3 and 4 show the comparisons between the two simulations (the solid lines are related to the semi-implicit scheme, and the dotted lines to the explicit one). The elevations (fig. 3) and current roses (fig. 4) are compared at four reference points.

The analysis of these figures shows that the elevations are identical and the velocities very similar. As the computation time is five times

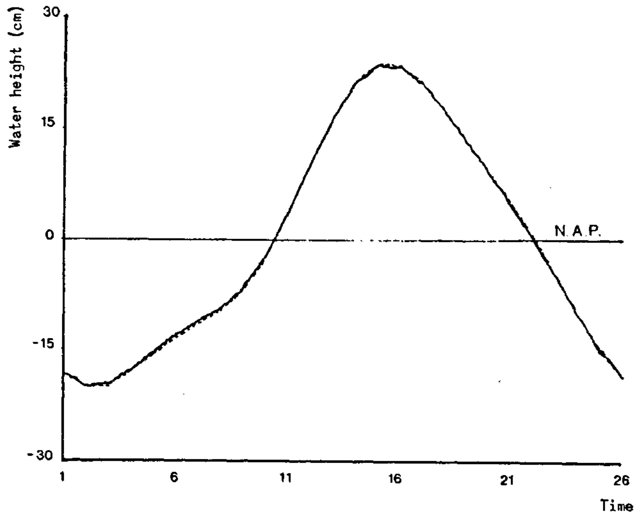

fig. 3a.
Sea elevations for station A

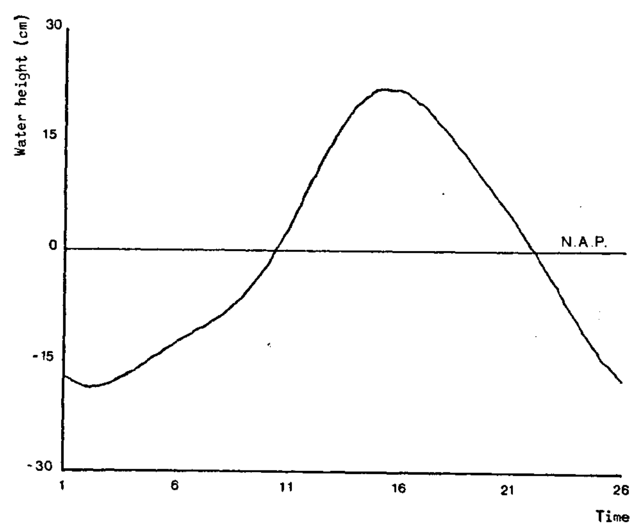

fig. 3b.
Sea elevations for station B

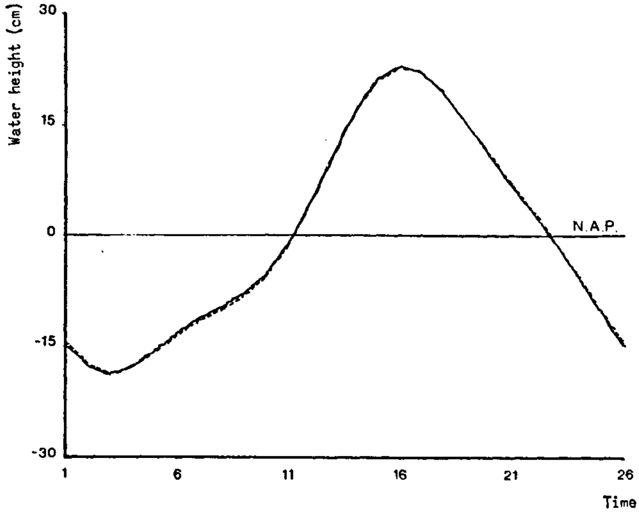

fig. 3c.

Sea elevations for station C

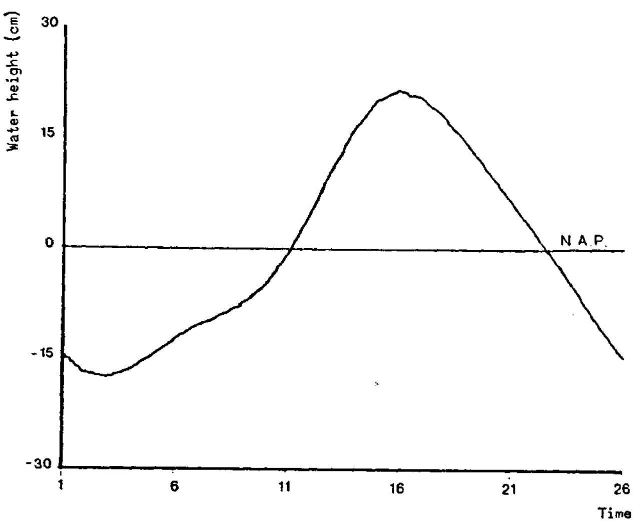

fig. 3d.

Sea elevations for station D

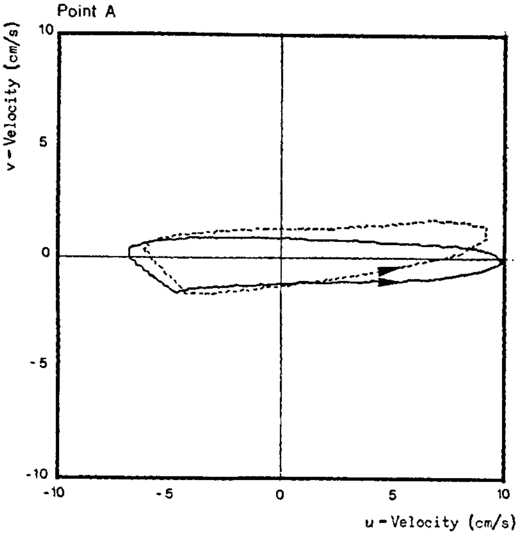

fig. 4a.

Current roses

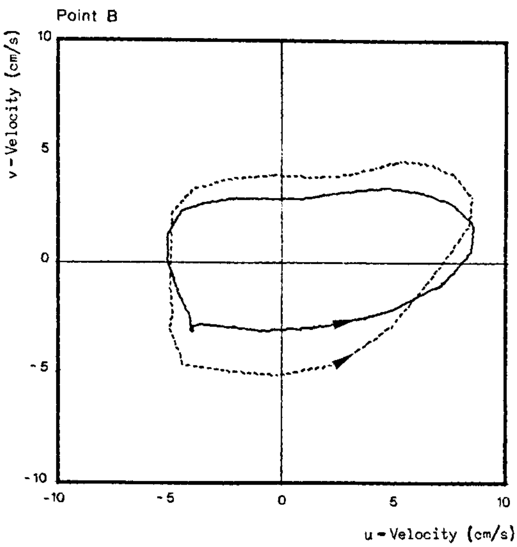

fig. 4b.

Current roses

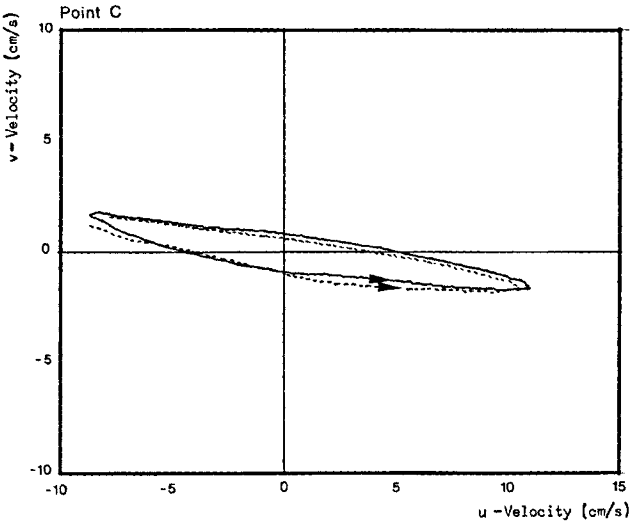

fig. 4c.

Current roses

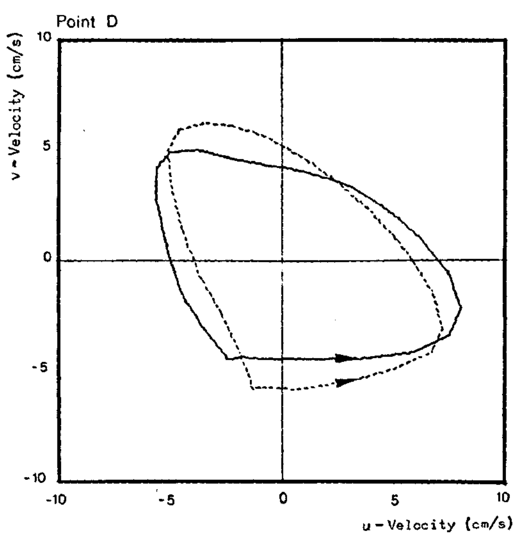

fig. 4d.

Current roses

higher with the explicit code and the quality of the results rather equal, one can conclude that the semi-implicit model is the most economical. Unfortunately, this code cannot be easily adapted to areas where sand banks are merging at low tide.

Conclusions

To study the prediction of dynamic processes in relation with coastal engineering projects and the generation of tidal harmonics, one must apply a predictor-corrector procedure. Unfortunately, it is very expensive when the refinement of the bottom topography is required. This technique is also adapted for areas where sand banks are merging at low tide.

For deeper areas, the simple semi-implicit model is very convenient and inexpensive if the bottom topography is not too irregular.

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